Exposure Assessment: Tolerance Limits, Confidence Intervals and Power Calculation based on Samples with Multiple Detection Limits

K. Krishnamoorthy[1], Thomas Mathew[2] and Zhao Xu[1]

[1]Department of Mathematics, University of Louisiana at Lafayette

Lafayette, LA 70508-1010, USA

[2]Department of Mathematics and Statistics, University of Maryland Baltimore County

Baltimore, MD 21250, USA

1   Introduction

Suppose we have a sample of n exposure measurements with k detection limits, say, $DL_1, ..., DL_k$ from a workplace. The problems of interest in exposure assessment are (i) construction of upper confidence limits for an upper percentile of the exposure distribution and (ii) confidence interval for the mean of the exposure distribution. Let us assume that exposure measurements follow a lognormal distribution with parameters μ and σ. Then log-transformed exposure measurements follow a normal distribution with mean μ and standard deviation (SD) σ. Because of this one-to-one relation between these distributions, inference for a lognormal distribution can be obtained from those for a normal distribution. This relation between these two distributions leads to the following relation between the percentiles: The $100p$ percentile of the normal distribution is

$$\xi_p = \mu + z_p \sigma, \tag{1}$$

and the one for lognormal distribution is

$$\exp(\xi_p) = \exp(\mu + z_p \sigma), \tag{2}$$

where z_p is the $100p$ percentile of the standard normal distribution. Thus, the problem of finding an upper confidence limit for a lognormal percentile is equivalent to the one of finding upper confidence limit for the corresponding normal percentile. The mean of the normal distribution is simply μ where as the mean of the lognormal distribution is given by

$$\exp\left(\mu + \frac{\sigma^2}{2}\right). \tag{3}$$

Note that there is no one-to-one relation between the means of these distributions, and the mean of the lognormal distribution also depends on the SD σ of the normal distribution. Figure 1 shows the probability density plots of the lognormal distribution and normal distribution when $\mu = 2$ and $\sigma = 1$.
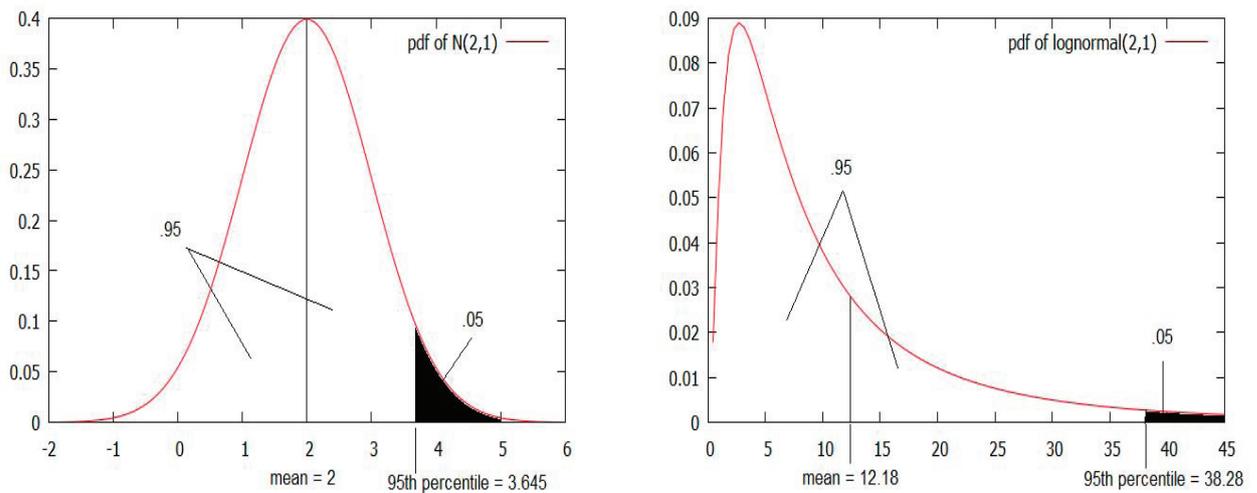
Figure 1: Probability density plots of $N(2,1)$ and lognorma$(2,1)$

The R program posted at "www.ucs.louisiana.edu/~kxk4695" calculates

1. The maximum likelihood estimates (MLEs) of μ and σ based on log-transformed exposure measurements with multiple detection limits (DLs),

2. upper confidence limit for an upper percentile of an exposure distribution (also known as the upper tolerance limit),

3. power of the test for
$$H_0 : \xi_p \geq \xi_p^0 \quad \text{vs.} \quad H_a : \xi_p < \xi_p^0,$$
where ξ_p^0 is a specified value of ξ_p, and

4. confidence intervals for the mean of an exposure distribution.

In the following sections, we shall briefly explain the methods that were used to obtain solutions to the aforementioned problems.

NOTATIONS

Symbols	Description
z_p	The $100p$ percentile of the standard normal distribution
Φ	Distribution function of the standard normal random variable
$\exp(\mu)$	The geometric mean of a lognormal distribution
$\exp(\sigma)$	The geometric standard deviation of a lognormal distribution
$\xi_p; \xi_p^0$	The $100p$ percentile of the $N(\mu, \sigma^2)$ distribution;
	$\xi_p = \mu + z_p\sigma$; ξ_p^0 is a specified value of ξ_p; e.g., $\xi_p^0 = \ln(OEL)$
DL_i	The detection limit for the ith laboratory method
	or sampling device; $i = 1, ..., k$
p_i	Probability of a nondetect for the ith device or the method; $i = 1, ..., k$
m_i	The number of non-detects below DL_i; $i = 1, ..., k$
m	$\sum_{i=1}^{k} m_i$, the total number of non-detects
$(\widehat{\mu}, \widehat{\sigma})$	The MLE of (μ, σ) based on a log-transformed random sample with
	log-transformed detection limits $\ln DL_1, ..., \ln DL_k$
$t_{m;1-\alpha}(\delta)$	$100(1 - \alpha)$ percentile of the noncentral t distribution with
	degrees of freedom m and the noncentrality parameter δ.
$\delta^* = \frac{\xi_p^0 - \xi_p}{\sigma}$	The scaled difference between the specified value and the true
	value of the $100p$ percentile of log-transformed exposure
	distribution (δ^* is independent of the unit of measurements)

2 Description of Methods

2.1 The Maximum Likelihood Estimates

Consider a simple random sample of n observations subject to k detection limits, say, $DL_1,...,DL_k$, from a lognormal distribution with parameters μ and σ. We assume without loss of generality that $DL_1 < DL_2 < ... < DL_k$. Further, we assume that detection limit DL_i for the ith laboratory or device can be expressed in the same measurement unit as that of detected observations. Suppose m_i nondetects are below DL_i, and let $m = \sum_{i=1}^{k} m_i$. Let $x_1, ..., x_{n-m}$ denote the log-transformed detected observations. Let

$$\bar{x}_d = \frac{1}{n-m} \sum_{i=1}^{n-m} x_i \quad \text{and} \quad s_d^2 = \frac{1}{n-m} \sum_{i=1}^{n-m} (x_i - \bar{x}_d)^2, \tag{4}$$

where $x_1, ..., x_{n-m}$ are detected observations. The log-likelihood function, after omitting a constant term, can be written as

$$l(\mu, \sigma) = \sum_{i=1}^{k} m_i \ln \Phi(z_i^*) - (n-m) \ln \sigma - \frac{(n-m)(s_d^2 + (\bar{x}_d - \mu)^2)}{2\sigma^2}, \tag{5}$$

where $z_i^* = \frac{\ln DL_i - \mu}{\sigma}$, $i = 1, ..., k$. The maximum likelihood estimates (MLES) $\widehat{\mu}$ and $\widehat{\sigma}$ are the values of μ and σ that maximize $l(\mu, \sigma)$.

The R program mles.dls(dl, x, indx) calculate the MLEs based on exposure measurements x(1),....,x(n) with detection limits dl(1),...,dl(k) and the vector "indx" of indicator variables; indx(i) = 1 if x(i) is censored, 0 otherwise. This program is based on the bivariate Newton-Raphson method. More details on usage of this program and an illustrative example are given in Section 4. For more technical details, see Krishnamoorthy and Xu (2011).

2.2   Confidence Limits for an Upper Percentile of an Exposure Distribution

If the exposure measurements follow a lognormal distribution, then the $100p$ percentile is given by

$$\exp(\mu + z_p \sigma). \tag{6}$$

As the log-transformed measurements follow the $N(\mu, \sigma^2)$ distribution, it is enough to find an uuper confidence limit for $\mu + z_p \sigma$. Note also that a $100(1-\alpha)$ upper confidence limit for the above percentile is referred to as the $(p, 1 - \alpha)$ upper tolerance limit.

The upper confidence limit for $\mu + z_p \sigma$ is constructed of the form

$$\widehat{\mu} + Q_{p;1-\alpha}^* \widehat{\sigma}, \tag{7}$$

where $\widehat{\mu}$ and $\widehat{\sigma}$ are the MLEs and $Q_{p;1-\alpha}^*$ is the tolerance factor determined so that

$$P\left(\widehat{\mu} + Q_{p;1-\alpha}^* \widehat{\sigma} \geq \xi_p\right) = 1 - \alpha.$$

Krishnamoorthy and Xu (2011) showed that the distribution of

$$\frac{\xi_p - \widehat{\mu}}{\widehat{\sigma}}$$

with detection limits, say, $L_1, ..., L_k$ is approximately the same as the distribution of

$$\frac{z_p - \widehat{\mu}^*}{\widehat{\sigma}^*}, \tag{8}$$

where $\widehat{\mu}^*$ and $\widehat{\sigma}^*$ are the MLEs based on a sample of size n from a standard normal distribution with detection limits $L_i^* = (L_i - \widehat{\mu})/\widehat{\sigma}$, $i = 1, ..., k$. That is, $\widehat{\mu}^*$ and $\widehat{\sigma}^*$ are the values of μ and σ, respectively, that maximize the log-likelihood function in (5) with $x_1, ..., x_{n-m}$ being a sample from a $N(0, 1)$ distribution with detection limits L_i^*. Thus, the factor $Q_{p;1-\alpha}^*$ is approximated by the $100(1 - \alpha)$ percentile of the distribution of $(z_p - \widehat{\mu}^*)/\widehat{\sigma}^*$. If the exposure measurements follow a lognormal distribution, then $L_i = \ln(DL_i)$ and $L_i^* = (\ln(DL_i)^* - \widehat{\mu})/\widehat{\sigma}$.

For given detection limits, the MLEs $\widehat{\mu}$ and $\widehat{\sigma}$, and the confidence level $1 - \alpha$, the the R function tol.limits(nr, n, dl, uh0, sh0, p, clev) calculates upper confidence limit U for $\mu + z_p\sigma$; this upper confidence limit is also known as the $(p, 1 - \alpha)$ upper tolerance limit for the normal distribution. Note that $\exp(U)$ is an upper confidence limit for the $100p$ percentile of the lognormal exposure distribution. More calculation details with an illustrative example are given in Section 4.

2.3 Power Calculation

Suppose we wish to test the hypothesis that the $100p$th percentile of the exposure distribution is below an occupational exposure limit (OEL). Formulating this as the alternative hypothesis, we thus have the following null hypothesis H_0 and alternative hypothesis H_a:

$$H_0 : \exp(\xi_p) \geq \exp(\xi_p^0) \;\; \text{vs.} \;\; H_a : \exp(\xi_p) < \exp(\xi_p^0) \;\; \Longleftrightarrow \;\; H_0 : \xi_p \geq \xi_p^0 \;\; \text{vs.} \;\; H_a : \xi_p < \xi_p^0, \quad (9)$$

where $\xi_p^0 = \ln(OEL)$, and ξ_p is the log-transformed $100p$th percentile of the exposure distribution. The test on the basis of the upper confidence limit in (7) rejects the null hypothesis in (9) when

$$\widehat{\mu} + Q_{p;1-\alpha}^*\widehat{\sigma} < \xi_p^0, \;\; \text{equivalently,} \;\; \frac{\xi_p^0 - \widehat{\mu}}{\widehat{\sigma}} > Q_{p;1-\alpha}^*. \quad (10)$$

Following the arguments of Krishnamoorthy and Xu (2011), the power function can be expressed as

$$P\left(\frac{\xi_p^0 - \widehat{\mu}}{\widehat{\sigma}} > Q_{p;1-\alpha}^*\right) \simeq P\left[\frac{\delta^* + z_p - \widehat{\mu}^*}{\widehat{\sigma}^*} > Q_{p;1-\alpha}^*\right], \quad (11)$$

where

$$\delta^* = (\xi_p^0 - \xi_p)/\sigma,$$

and $(\widehat{\mu}^*, \widehat{\sigma}^*)$ is as defined in (8). It is clear from the power function in (11) that the power of the test depends on the parameters only via $\delta^* = (\xi_p^0 - \xi_p)/\sigma$. However, it is difficult to derive the power function explicitly, and simulation studies should be carried out to understand its power properties. The approximate power function on the right-hand side of (11) can be estimated using the following Algorithm 1.

Algorithm 1

For given values of $(n, p, \alpha, DL_1, ..., DL_k, \delta^*, \xi_p^0, \sigma)$:

1. Calculate z_p and $\mu = \xi_p^0 - (\delta^* + z_p)\sigma$ and set $DL_i^* = \frac{\ln DL_i - \mu}{\sigma}, \;\; i = 1, ..., k$.

2. Generate a sample of size n from a $N(0, 1)$ distribution with detection limits $DL_1^*, ..., DL_k^*$.

3. Compute the MLEs $\widehat{\mu}^*$ and $\widehat{\sigma}^*$ based on the generated sample in step 2. Set $Q_p^* = (z_p - \widehat{\mu}^*)/\widehat{\sigma}^*$ and $Q = (\delta^* + z_p - \widehat{\mu}^*)/\widehat{\sigma}^*$.

4. Repeat steps 2 and 3 for 10,000 times.

5. Find the $100(1-\alpha)$ percentile of 10,000 Q_p^*'s, and call it $Q_{p;1-\alpha}^*$. The percentage of these 10,000 Q's that are greater than $Q_{p;1-\alpha}^*$ is an estimate of the approximate power on the right-hand side of (11).

Our extensive simulation studies indicated that the approximate powers are very close to the exact ones on the left-hand side of (11).

The R function power.calc.uncens(n, p, delts, alpha) calculates the exact power of the test based on samples with no detection limit. power.calc.cens(nr,n,dl,p,delts,xip0,sig,alpha) calculates the approximate power in (11) based on samples with multiple detection limits.

2.4 Confidence Intervals for a Lognormal Mean

As the mean of a lognormal distribution with parameters μ and σ^2 is given by $\exp(\mu + .5\sigma^2)$, it is enough to find a confidence interval for $\eta = \mu + .5\sigma^2$, where μ and σ^2 are respectively the mean and variance of a normal distribution. A confidence interval for η can be obtained using the generalized variable approach given in Krishnamoorthy and Xu (2011). Let $\widehat{\mu}_0$ and $\widehat{\sigma}_0$ be observed values of the MLEs based on a sample of n observations with k detection limits. An approximate *generalized pivotal quantity* for η, denoted by G_η, is given by

$$G_\eta = \widehat{\mu}_0 - \frac{\widehat{\mu}^*}{\widehat{\sigma}^*}\widehat{\sigma}_0 + .5\frac{\widehat{\sigma}_0^2}{\widehat{\sigma}^{*2}}, \tag{12}$$

where $\widehat{\mu}^*$ and $\widehat{\sigma}^*$ are as defined in (8). For a given sample size n, detection limits and the MLEs $(\widehat{\mu}_0, \widehat{\sigma}_0)$, Monte Carlo simulation can be used to estimate the percentiles of G_η. A confidence interval or one-sided confidence limit for η based on the generalized pivotal quntity G_η can be evaluated using Algorithm 2.

Algorithm 2

For a given data set with k detection limits, compute the MLEs $\widehat{\mu}_0$ and $\widehat{\sigma}_0$,

1. Set $DL_i^* = \frac{\ln DL_i - \widehat{\mu}_0}{\widehat{\sigma}_0}$, $i = 1, ..., k$.

2. Generate a sample of size n from $N(0,1)$ distribution with detection limits $DL_1^*, ..., DL_k^*$.

3. Compute the MLEs $\widehat{\mu}^*$ and $\widehat{\sigma}^*$ based on the sample generated above.

4. Set $G_\eta = \widehat{\mu}_0 - \frac{\widehat{\mu}^*}{\widehat{\sigma}^*}\widehat{\sigma}_0 + .5\frac{\widehat{\sigma}_0^2}{\widehat{\sigma}^{*2}}$.

5. Repeat steps 2–5 for a large number of times, say, 10,000.

The $100(1-\alpha)$ percentile of 10,000 G_η's (denoted by $G_{\eta;1-\alpha}$) is the $(p, 1-\alpha)$ upper confidence limit for η. The interval $\left(G_{\eta;\frac{\alpha}{2}}, G_{\eta;1-\frac{\alpha}{2}}\right)$ is a $1-\alpha$ confidence interval of η. The R function ci.logmean(nr,

n, dl, uh0, sh0, clev) calculates one-sided confidence limit as well as two-sided confidence interval for a lognormal mean.

3 R program

1. If you have not already installed R in your computer, download R package from http://www.r-project.org/, and install it.
2. Download the R source file EXPOSURE_MDLS.r from 'www.ucs.louisiana.edu/~kxk4695' and save it in a directory, say, MYDIR.
3. Open R, click 'File --> Source R code' and then locate the R file EXPOSURE_MDLS.r that you saved.

You will see the following line at the command prompt '>' .

```
> source("C://MYDIR//EXPOSURE_MDLS.r")
```

Now all function routines are loaded to compute the MLEs, upper confidence limits for a percentile, confidence intervals for a lognormal mean, and power calculation for a test on an upper percentile.

```
1. mles.dls(dl, x, indx): to compute the MLEs for a given sample x
                          with detection limits dl(1),...,dl(m);
                          indx(i) = 1 if x(i) is censored; 0 otherwise

2. tol.limits(nr, n, dl, uh0, sh0, p, clev): to compute 100clev% upper
                          confidence limit for the 100p percentile of the
                          exposure distribution based on the MLEs "uh0" of
                          the mean and "sh0" of the standard deviation.
          nr = number of simulation runs;
          n = sample size;
          uh0 = MLE of the mean;
          sh0 = MLE of the std deviation;
          p = content level of the tolerance limit (p = .90 means 90th percentile
              of exposure distribution)
          clev = confidence level of the tolerance limit (OR confidence level of
                  the upper confidence limit for 100pth percentile)
3. ci.logmean(nr, n, dl, uh0, sh0, clev): to compute one-sided upper
                          confidence limits and two-sided confidence
                          intervals for a lognormal mean.
4. power.calc.uncens(n, p, delts, alpha): to compute the power
                          of a test on a percentile of a lognormal distribution
                          based on an uncensored sample.
```

```
5. power.calc.cens(nr,n,dl,p,delts,xip0,sig,alpha): to compute the power
                             of a test on a percentile of a lognormal distribution
                             based on a censored sample.
```

3.1   Importing Data

Suppose you have a sample of data from a lognormal distribution with two detection limits .47 and 1.13 as in the following table.

Table 1. Sample Data;							
<0.47	<0.47	0.78	1.10	<1.13	<1.13	<1.13	1.36

The data file should be in the following format:

Step 1: Arrange data in a txt file, say, sample.txt, in two columns.

0.47   1
0.47   1
0.78   0
1.10   0
1.13   1
1.13   1
1.13   1
1.36   0

The first column contains the measurements, replacing each observation below detection limit by the corresponding detection limit, and the second column is the index indicating whether the observation is a detect or non-detect, 1 represents non-detect, 0 represents detect.

Step 2: Save the above text file in your computer, say, drive F, then input the command in R as follows:

> xdata <- read.table("F:/sample.txt",header = FALSE)
> x <- xdata[,1]
> indx <- xdata[,2]

Then, the vector "x" is the sample data, and the vector "indx" is the index indicating detect or nondetect.

You can also directly input the data in R as follows:

```
> x <- c(0.47,0.47,0.78,1.10,1.13,1.13,1.13,1.36)
> indx <- c(1,1,0,0,1,1,1,0)
```

```
> dl <- c(.47, 1.13)
```

Remember the data in the vectors "x" and "indx" should be paired to each other.

To compute the MLEs for the data in Table 1:

```
> x <- c(0.47,0.47,0.78,1.10,1.13,1.13,1.13,1.36)
> indx <- c(1,1,0,0,1,1,1,0)
> dl <- c(.47, 1.13)
> x <- log(x)
> dl <- log(dl)
> mles.dls(dl,x,indx)
[1] -0.5391601  0.6205773 #the first entry is the mle of mu; the 2nd is the mle of sigma
```

4   Illustrative Example for Computing the MLEs, Tolerance Limits, and Confidence Intervals

Example 2:   Consider the following sample with three detection limits ($DL_1 = .47$, $DL_2 = 1.13$ and $DL_3 = 3.62$) from a lognormal distribution.

Table 1: Simulated data from a lognormal distribution

< 0.47	< 0.47	0.78	1.10	< 1.13	< 1.13	< 1.13	1.36	
1.54	1.67	2.30	2.71	< 3.62	< 3.62	< 3.62	< 3.62	
< 3.62	< 3.62	< 3.62	< 3.62	5.78	7.30	15.26	17.43	28.38

The following is a snapshot of the R window where the MLEs are computed; the first number is the MLE of μ, and the second one is the MLE of σ. The data and other input values can be entered manually as shown below. First, start R, click

```
'File --> Source R code' and then locate the R file EXPOSURE_MDLS.r that you saved.
```

```
> source("C:\\MYDIR\\EXPOSURE_MDLS.r")
> x <-    c(.47, .47, .78,1.10,1.13,1.13,1.13,1.36,1.54,1.67, 2.30, 2.71,
+          3.62,3.62,3.62,3.62,3.62,3.62,3.62,3.62,5.78,7.30,15.26,17.43, 28.38)
> indx <- c(1,    1,   0,   0, 1,   1,     1,   0,   0,   0,    0,    0,
+            1,    1,   1,   1,   1,   1,   1,   1,   0,   0,    0,    0,     0)
> dl <- c(.47,1.13,3.62)
> x <- log(x)
> dl <- log(dl)
> mles.dls(dl, x, indx)
[1] 0.2292267 1.5371946
```

That is, .2292267 is the MLE of μ and the second entry 1.5371946 is the MLE of σ.

The following snapshot of R window describes computation of (.90, .95) upper tolerance limit based on the MLEs calculated earlier.

```
> nr <- 10000
> n <- 25
> uh0 <- .229  #from the earlier calculation
> sh0 <- 1.537
> dl <- log(c(.47,1.13,3.62))
> p <- .90
> clev <- .95
> tol.limits(nr,n,dl,uh0,sh0,p,clev)
[1] "Tolerance Factor"
[1] 1.94911*
[1] "-----------------------------"
[1] "Upper Tolerance Limit"
[1] 25.14809* # which is exp(.229 + 1.94911 x 1.537)
```

[*when you run the program you may not get the same numbers, because the results are based on simulation; you should get results very close to the above ones]

The following snapshot of the R window describes calculation of the 95% one-sided upper confidence limit, and 95% two-sided confidence interval for the mean of the lognormal distribution based on the MLEs calculated earlier.

```
> nr <- 10000
> n <- 25
> uh0 <- .229  # the mle calculated earlier
> sh0 <- 1.537 # the mle calculated earlier
> dl <- log(c(.47,1.13,3.62))
> clev <- .95  # confidence level
> ci.logmean(nr,n,dl,uh0,sh0,clev)
[1] "----------------------------------"
[1] "upper confidence limit "
[1] 17.41529*
[1] "----------------------------------"
[1] "two-sided confidence interval"
[1]  2.080411* 26.276086*
>
```

[*when you run the program you may not get the same numbers, because the results are based on simulation; you should get results very close to the above ones]

5　An Illustrative Example for Power Calculation

Example 3: The OSHA permissible exposure limit (PEL) for carbon monoxide is 50 ppm[1]. It is desired to test if the 95th percentile of the contaminant distribution is indeed less than 50 ppm. In the notations of this paper, the hypotheses of interest are

$$H_0 : \xi_{.95} \geq \xi_{.95}^0 \quad \text{vs.} \quad H_a : \xi_{.95} < \xi_{.95}^0, \tag{13}$$

where $\xi_{.95}$ is the true log-transformed 95th percentile of the contaminant distribution in the workplace, and $\xi_{.95}^0 = \ln(50) = 3.912$. An industrial hygienist, based on his preliminary inspection, guesstimates that the 95th percentile of the concentration distribution in his workplace is around 20 ppm; that is $\xi_{.95} = \ln(20)$ It is desired to determine the sample size so that the test would reject H_0 in (13) with probability (power) .90, when the true 95th percentile is 20ppm. On the basis of past data or based on data from a similar workplace, suppose the value of the geometric SD $\exp(\sigma)$ is guessed to be 2.015ppm. Then

$$\delta^* = (\xi_p^0 - \xi_p)/\sigma = (\ln(50) - \ln(20))/\ln(2.015) = 1.308.$$

Since we are computing the power at the value $\xi_{.95} = \ln(20)$, $\mu = \ln(20) - z_{.95}\sigma = \ln(20) - 1.645 \times \ln(2.015) = 1.843$. In other words, the power is to be calculated at the parameter values $\xi_{.95} = \ln(20)$, $\sigma = \ln(2.015)$, or equivalently,

$$\mu = 1.843, \text{ and } \sigma = \ln(2.015) = .701.$$

For this particular example, if no nondetect is expected, then the required sample size n is determined by the following equation.

$$P\left(t_{n-1}((\delta^* + z_{.95})\sqrt{n}) > t_{n-1;.95}(z_{.95}\sqrt{n})\right) = .90 \iff P\left(t_{n-1}(2.954\sqrt{n}) > t_{n-1;.95}(1.645\sqrt{n})\right) = .90.$$

The sample size can be determined as follows.

```
> p <- .95 #the percentile of the exposure distribution to be tested
> delts <- 1.309 # the value of delat-star
> alpha <- .05 # the level of the test
> power.calc.uncens(18,p,delts,alpha)
[1] 0.867301
> power.calc.uncens(19,p,delts,alpha)
[1] 0.8871927
> power.calc.uncens(20,p,delts,alpha)
[1] 0.9044049
```

---

[1] http://www.osha.gov/pls/oshaweb/owadisp.show_document?p_table=standards&p_id=9992

So if no nondetect is expected, then the required sample size to attain the power of .90 is 20.

Suppose the samples will be analyzed by three laboratory methods or devices with the detections limits $(\ln DL_1, \ln DL_2, \ln DL_3) = (1.3, 1.5, 1.7)$. Then the sample sizes and powers can be calculated as follows.

```
> nr <- 10000
> dl <- c(1.3,1.5,1.7)
> delts <- 1.309
> p <- .95
> alpha <- .05
> xip0 <- 3.912
> sig <- .701
> power.calc.cens(nr,21,dl,p,delts,xip0,sig,alpha) # power at n=21
[1] 0.849
> power.calc.cens(nr,22,dl,p,delts,xip0,sig,alpha) # power at n=22
[1] 0.873
> power.calc.cens(nr,23,dl,p,delts,xip0,sig,alpha) # power at n=23
[1] 0.880
> power.calc.cens(nr,24,dl,p,delts,xip0,sig,alpha) # power at n=24
[1] 0.891
> power.calc.cens(nr,25,dl,p,delts,xip0,sig,alpha) # power at n=25
[1] 0.909
>
```

So a sample of size 25 is required to attain a power of .90.