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LETTER

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Let $Y_{i1}, ..., Y_{in_i}$ be a sample from a lognormal distribution with parameters μ_i and σ_i, $i = 1, 2$. Assuming that both samples are independent, Singhasomboona, Panichkitkosolkula and Volodin (2020) have addressed the problem of interval estimating the ratio of the medians of two lognormal distributions. Noting that the ratio of medians is given by

$$\frac{\exp(\mu_1)}{\exp(\mu_2)} = \exp(\mu_1 - \mu_2), \tag{1}$$

we see that the problem simplifies to interval estimation of $\mu_1 - \mu_2$. Furthermore, letting $X_{ij} = \ln Y_{ij}$, $j = 1, ..., n_i$, $i = 1, 2$, we see that $X_{i1}, ..., X_{in_i} \sim N(\mu_i, \sigma_i^2)$, $i = 1, 2$, and so the problem is further simplified to find confidence intervals (CIs) for the difference between two normal means when the variances are unknown and arbitrary. This problem is one of the well-known classical problems, and is referred to as the Behrens-Fisher (B-F) problem. Therefore, any solution for the B-F problem can be used to find a CI for the ratio of the medians of two lognormal distributions.

Let $(\overline{X}_i, S_i^2)$ denote the (mean, variance) based on $X_{i1}, ..., X_{in_i}$, $i = 1, 2$. One of the best CI for $\mu_1 - \mu_2$ is known as the Welch approximate degrees of freedom CI (Welch 1947). The $(1 - \alpha)$ Welch CI is given by

$$(L, U) = \overline{X}_1 - \overline{X}_2 \pm t_{\nu;1-\alpha/2}\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}, \text{ where } \nu = \frac{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^2}{\frac{S_1^4}{n_1^2(n_1-1)} + \frac{S_2^4}{n_2^2(n_2-1)}}. \tag{2}$$

The above CI is described in many introductory level text books (e.g., McClave and Sincich 2013) and can be computed using calculators such as TI-84. The $(1 - \alpha)$ CI for the ratio of medians in (1) is $(\exp(L), \exp(U))$. We refer to this CI for the ratio of the medians as the Welch CI.

I carried out some simulation studies to compare the performance of the Welch CI with the generalized CI (GCI) that has better coverage probabilities than other CIs; see Singhasomboona et al. (2020). I used 100,000 simulated CIs to estimate the coverage probability (CP) and average length (AL) of the Welch CI. To estimate the CP and AL of the generalized CI, I used 10,000 CIs, each was estimated using 100,000 simulation runs. The CPs and ALs are reported in the following table for some small to moderate values of sample sizes.

We observe from Table 1 that the Welch CI has better coverage probabilities with smaller average lengths for all the cases. Thus, not only the Welch CI is in closed-form and simple to compute, but also better than the GCI.

Table 1. Coverage probabilities and average lengths of 95% confidence intervals.

	$(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2)$					
	(1,1,.2,.2)		(1,1,.2,.5)		(2,1,.4,1)	
(n_1, n_2)	Welch	GCI	Welch	GCI	Welch	GCI
(5, 5)	.956(1.486)	.976(1.797)	.953(2.223)	.972(2.597)	.953(10.70)	.975(13.52)
(10, 5)	.949(1.215)	.967(1.369)	.953(1.476)	.971(1.705)	.952(6.442)	.971(7.453)
(10, 10)	.951(0.879)	.963(0.933)	.951(1.221)	.962(1.286)	.949(5.138)	.957(5.399)
(10,15)	.950(0.790)	.963(0.840)	.952(1.014)	.964(1.071)	.951(4.172)	.963(4.421)
(15,15)	.952(0.686)	.960(0.713)	.950(0.936)	.961(0.969)	.951(3.815)	.959(3.968)
(10,25)	.950(0.724)	.961(0.754)	.951(0.857)	.961(0.902)	.951(3.468)	.963(3.610)
(25,25)	.949(0.516)	.956(0.529*)	.951(0.695)	.953(0.714)	.951(2.764)	.953(2.820)

The value of .4551 reported in Table 1 of Singhasomboona et al. (2020) appears to be in error. My simulation studies indicated that the average lengths for GCIs reported in Tables 1 and 3 of Singhasomboona et al. (2020) are all inaccurate while the average lengths in their Table 2 are in agreement with my results.

References

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