$$E = (\underline{e}_1, \underline{e}_2)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\underline{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$F = (\underline{f}_1, \underline{f}_2)\begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix}$$

$[x]_E$ $\qquad\qquad$ $[x]_F$

$$[x]_E \underset{F^{-1}}{\overset{F}{\rightleftarrows}} [x]_F$$

$[x]_E = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

$[x]_F$

Left multiply $[x]_E$ by F^{-1} to get $[x]_F$

Left multiply $[x]_F$ by F to get $[x]_E$

GVSM

$$A = \left(\underline{d}_1^T \; \underline{d}_2^T \; \underline{d}_3^T \cdots \underline{d}_n^T \right)$$

$$\underline{d}_\alpha = (a_{1\alpha}, a_{2\alpha}, \cdots a_{n\alpha})$$

$$A^T = \begin{pmatrix} \underline{d}_1 \\ \underline{d}_2 \\ \underline{d}_3 \\ \vdots \\ \underline{d}_n \end{pmatrix}$$

We treat A^T as the basis of vector space

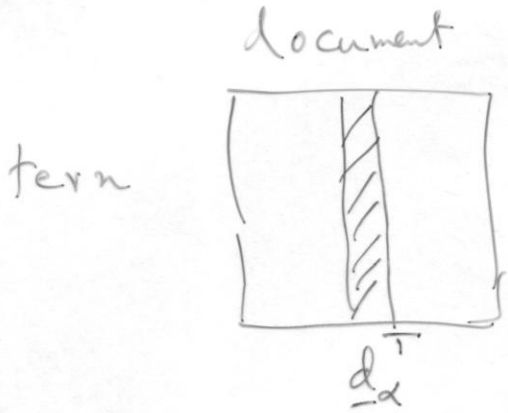
$$[\underline{d}_\alpha]^T_{A^T}$$

$$[\underline{d}_\alpha]^T_E = A^T [\underline{d}_\alpha]^T_{A^T}$$

$$[\underline{q}]^T_E = A^T [\underline{q}]^T_{A^T}$$

$$[d_\alpha]_E \cdot [q]^T_E = \left(A^T [\underline{d}_\alpha]^T_{A^T}\right)^T \left(A^T [\underline{q}]^T_{A^T}\right)$$

$$= [d_\alpha]^T_{A^T} A A^T [\underline{q}]^T_{A^T}$$



$[\underline{d}_\alpha]^T_{A^T}$ is given by columns of the term - document matrix

G_t is given by

$\underline{q}$ is a row vector of coefficients of the query vector.

$$\underline{d}_\alpha \cdot \underline{q} = [\underline{d}_\alpha]_{A^T} G_t [\underline{q}^T]_{A^T}$$

$$RSV_q^T = \begin{pmatrix} rsv(d_1) \\ rsv(d_2) \\ \vdots \\ rsv(d_m) \end{pmatrix} = A^T G_t \underline{q}^T \quad AA^T$$

$$[\underline{d}_\alpha]_{A^T} A A^T [\underline{q}^T]_{A^T}$$

$n = 2$

$$A = \begin{array}{cc} \underline{d}_1 & \underline{d}_2 \\ \downarrow & \downarrow \\ \end{array}$$
$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

$$A^T = \begin{array}{cc} \underline{t}_1^T & \underline{t}_2^T \\ \end{array}$$
$$\begin{pmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{pmatrix}$$

$$[\underline{d}_1]_E^T = A^T \begin{pmatrix} a_{11} \\ a_{21} \end{pmatrix} \leftarrow [\underline{d}_1]^T_{A^T}$$

$$[\underline{d}_2]_E^T = A^T \begin{pmatrix} a_{12} \\ a_{22} \end{pmatrix} \leftarrow [\underline{d}_2]^T_{A^T}$$

$$\rightarrow A A^T$$

$$[\underline{d}_\alpha]_{A^T} \overset{\text{def}}{=} G_t [\underline{q}^T]_{A^T}$$

$$A = \begin{pmatrix} 2.5 & 1.5 \\ 1.2 & 6.5 \end{pmatrix}$$

$$[\underline{d}_1]_{A^T} = (2.5 \quad 1.2)$$

$$[\underline{d}_2]_{A^T} = (1.5 \quad 6.5)$$

$$[\underline{q}]_{A^T} = (2 \quad 3)$$

$$RSV(\underline{d}_1, \underline{q})$$

$$\xrightarrow{GVSM} \quad G_t = AA^T = \begin{pmatrix} 2.5 & 1.5 \\ 1.2 & 6.5 \end{pmatrix}\begin{pmatrix} 2.5 & 1.2 \\ 1.5 & 6.5 \end{pmatrix}$$

$$RSV(d_1, q) = [\underline{d}_1]_{A^T} G_t [\underline{q}^T]_{A^T}$$

$$= (2.5 \quad 1.2)\, G_t \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$