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ELECTRIC FIELD DEPENDENCE OF MOBILITY FLUCTUATION 1/f NOISE IN ELEMENTAL SEMICONDUCTORS†

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Abstract—It is shown that Bosman's empirical formula $\alpha(E) = \alpha(0)/[1 + (E/E_c')^2]$ for the field dependence of Hooge's parameter can be explained if it is assumed that only the low-field value μ_{aco} of the acoustical mobility μ_{ac} fluctuates and that $\mu_{ac} = \mu_{aco}\,[1 + (E/E_c')^2]^{1/2}$.

Experimental investigations of the electric field dependence of Hooge's parameter[1] α have been carried out[2–4] and a number of empirical formulas have been quoted in the literature.

Bosman *et al.*[2] found

$$\alpha(E) = \frac{\alpha(0)}{1 + (E/E_c')^2} \tag{1}$$

where $\mu_0 E_c' \simeq s$; s is the speed of sound in the medium, $\alpha(0)$ the low field value of α and μ_0 is the low-field carrier mobility.

A theoretical justification for these results has not been given so far. We shall try to put this empirical formula on a simple theoretical basis.

Let us first look for a justification of eqn (1). If all measurements are made under open-circuit conditions, we have

$$\delta I = 0. \tag{2}$$

Now

$$I = q\mu n E \tag{3}$$

and, therefore, eqn (3) gives

$$\delta(\mu E) = 0. \tag{4}$$

Experimentally one finds for silicon

$$\mu = \frac{\mu_0}{1 + E/E_c} \tag{5}$$

where $\mu_0 E_c = u_c$ is the critical velocity of the carriers, which is about 10 times the speed of sound in the medium. If one assumes that the functional relation

$$E_c = u_c/\mu_0 \tag{6}$$

also holds for the fluctuations and that u_c does not fluctuate, we obtain for small fluctuations in μ_0

$$\frac{S_v(f)}{V^2} = \frac{S\mu_0^{(f)}}{\mu_0^2} \tag{7}$$

On the other hand, if E_c does not fluctuate

$$\frac{S_v(f)}{V^2} = \frac{S\mu_0^{(f)}}{\mu_0^2}(1 + E/E_c)^2. \tag{8}$$

Since eqns (7) and (8) do not agree with eqn (1) this approach does not work.

Let us, therefore, try the following approach. Let μ_{ac} be the mobility due to acoustical mode scattering and μ_{op} the mobility due to optical mode scattering, then

$$\frac{1}{\mu} = \frac{1}{\mu_{ac}} + \frac{1}{\mu_{op}}. \tag{9}$$

It has been shown at low fields that μ_{aco}, the low-field value of μ_{ac}, can have slow fluctuations[5]. Let us assume on the other hand, that μ_{op} does not fluctuate. In that case we may write

$$\mu = f(\mu_{aco}, E) \tag{10}$$

and obtain, if only μ_{aco} fluctuates

$$\frac{S_v(f)}{V^2} = \frac{1}{\left(1 + \dfrac{E}{\mu}\dfrac{\partial\mu}{\partial E}\right)^2}\left(\frac{\partial\mu}{\partial\mu_{aco}}\right)^2\left(\frac{\mu_{aco}}{\mu}\right)^2\frac{S\mu_{aco}(f)}{\mu_{aco}^2}. \tag{11}$$

Now writing[5]

$$\frac{S\mu_{aco}}{\mu_{aco}^2} = \frac{\alpha_1}{Nf} \tag{12}$$

where N is the number of carriers, f the frequency and α_1 a constant, and evaluating $\partial\mu/\partial E$ from (5) yields

$$\frac{S_v(f)}{V^2} = \left(\frac{\partial\mu}{\partial\mu_{aco}}\right)^2\left(\frac{\mu_{aco}}{\mu_0}\right)^2\left(1 + \frac{E}{E_c}\right)^4\frac{\alpha_1}{Nf}. \tag{13}$$

We now write

$$\mu_{ac} = \mu_{aco}\,g(E) \tag{14}$$

where $g(E)$ is a function of E that tends to unity for $E \to 0$. Substituting (14) and (9) into (13) gives

$$\frac{S_v(f)}{V^2} = \frac{\alpha(0)}{Nf}\frac{1}{g^2} \tag{15}$$

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if it is assumed that E'_c does not fluctuate. Here

$$\alpha(0) = \alpha_1(\mu_0/\mu_{aco})^2. \quad (15a)$$

We note that (15) corresponds to eqn (1) if

$$g = [1 + (E/E'_c)^2]^{1/2} \quad (16)$$

where $\mu_0 E'_c \simeq s$, the speed of sound in the medium. Equation (16) can be justified from the fact that at sufficiently high fields the carriers gain sufficient energy to emit optical phonons, even when drifting over relatively short distances. Hence, optical phonon scattering builds up at the expense of acoustical phonon scattering, so that $g(E)$ should indeed increase with increasing E.

However, if we write

$$E'_c = s/\mu_0 \quad (17)$$

and assume that E'_c fluctuates because μ_0 fluctuates through μ_{aco} we obtain

$$\frac{S_v(f)}{V^2} = \frac{\alpha(0)}{Nf}\frac{1}{1+(E/E'_c)^2} \left\{\frac{[1+(E/E'_c)^2]^{1/2} + \left(\frac{\mu_{opo}}{\mu_{aco}+\mu_{opo}}\right)\left(\frac{E}{E'_c}\right)^2}{1+(E/E'_c)^2}\right\} \quad (18)$$

The expression between brackets reduces to unity at low fields and to $[\mu_{opo}/(\mu_{aco}+\mu_{opo})]^2$ for $E/E'_c \gg 1$. Since μ_{opo} is probably much larger than μ_{aco}, this factor is close to unity at all fields, so that eqn (1) will be valid in good approximation.

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