Solid-State Electronics Vol. 24, No. 10, pp. 905–906, 1981
Printed in Great Britain.

0038–1101/81/100905–02$02.00/0

EFFECT OF TRANSVERSE ELECTRIC FIELD ON NYQUIST NOISE

R. P. JINDAL and A. VAN DER ZIEL

Electrical Engineering Department, University of Minnesota, Minneapolis, MN 55455, U.S.A.

(*Received* 3 *January* 1981; *in revised form* 3 *March* 1981)

Abstract—According to Nyquist's theorem, in the low frequency limit, the thermal noise voltage spectrum can be written as $S_v(f) = 4\,\text{kTR}$. Based on this, we can define, $R_{\text{Nyquist}} = S_v(f)/4\,\text{kT}$ as the Nyquist resistance of the sample. We can also define the observed zero bias resistance of a sample as $R_{\text{observed}} = \lim_{V\to o} (V/I)$. It is common practice to substitute R_{observed} for R_{Nyquist} in order to estimate the thermal noise. It will be shown that this substitution is not always valid in MOSFETs, and that the data obtained by Takagi and van der Ziel can be explained in this manner.

INTRODUCTION

Takagi and van der Ziel[1] have experimentally demonstrated that for a MOSFET at zero drain bias, the experimental measurements of thermal noise do not match with the theoretical estimates. It will be shown that this is due to the strong transverse electric field experienced by the MOSFET inversion channel and finite mobility gradient perpendicular to the oxide-semiconductor interface.

General analysis for an n channel MOSFET inversion layer.

It has been shown[2] that for a limited range, a MOSFET inversion layer can be divided into sublayers and one can specify a carrier density "n" and a mobility "μ" to each sublayer. Further, it has been shown[2] that due to intersublayer interaction, the effective mobility of a sublayer is given by

$$\mu_{\text{eff}} = \mu_0(x)\left\{1 - \frac{\mu_0(x)E_x(x)m}{q}\frac{\partial\mu_0(x)}{\partial x}\right\} \tag{1}$$

where all the symbols have their usual meaning; for example, m is the effective mass of the carriers. The field due to the gate bias acts in the $\hat{x}$ direction and the conduction in the channel is along the $\hat{y}$ direction, the problem being assumed to be invariant along the $\hat{z}$ direction. The oxide semiconductor interface constitutes the $x = 0$ plane. It is assumed that $E_x \gg E_y$. E_x produces a transverse effect on the conduction in the $\hat{y}$ direction. In order to calculate the total transverse effect we shall sum over the effect on each sublayer and hence arrive at the total transverse effect.

Let n_B = electron density in the bulk of the device ($x = \infty$), $n(x)$ = electron concentration at some point at a distance "x" from the interface, $U = q\psi/\text{kT}$ = normalized potential, U_s = normalized surface potential, i.e. at $x = 0$, and U_B = normalized Bulk potential, i.e. at $x = \infty$

Then, since the sample is in equilibrium in the $\hat{x}$ direction (no net current flow) we have, assuming Boltzmann statistics,

$$n(x) = n_i \exp(U), \quad n_B = n_i \exp(U_B). \tag{2}$$

Total sheet conductance

$$\sigma_s = q\int_0^\infty \mu_{\text{eff}}(n(x) - n_B)\,\text{d}x$$

$$= \sigma_{s0} - \Delta\sigma_s. \tag{3}$$

Using Eq. (1), we obtain,

$$\sigma_{s0} = q\int_0^\infty \mu_0(x)[n(x) - nB]\,\text{d}x \tag{4}$$

and

$$\Delta\sigma_s = m\int_0^\infty \mu_0^2(x)\frac{\text{d}\mu_0(x)}{\text{d}x}E_x(x)[n(x) - n_B]\,\text{d}x. \tag{5}$$

For a MOSFET with channel length L and width W, channel conductance

$$G_{\text{eff}} = \frac{W}{L}\cdot\sigma_s = \frac{W}{L}(\sigma_{s0} - \Delta\sigma_s). \tag{6}$$

It should be noted that this is an expression for the *measured* channel conductance since the effect of a finite drain bias and hence finite carrier drift has been taken into account as mutual sublayer interaction[2].

However, if we consider the channel conductance under strict equilibrium conditions, i.e. with zero drift velocity for each sublayer, the sublayer interaction does not exist and we have

$$G_{\text{Nyquist}} = \frac{W}{L}\sigma_{s0}. \tag{7}$$

Note again that this conductance cannot be measured since any application of the drain bias, however small, disturbs the strict equilibrium.

Hence it is seen that the effective conductance (measured conductance) is smaller than the equilibrium conductance (Nyquist conductance). Since, thermal noise is an equilibrium phenomena, it is governed by Nyquist conductance and hence

$I_{eq|measured} > I_{eq|estimated}$ (based on measured conductance).

This is precisely the result that has been reported.

Detailed evaluation for the MOSFET

Let the discrepancy between the measured and estimated values of equivalent noise current be ΔI_{eq}. Then

$$\Delta I_{eq} = \frac{2kT}{q} \Delta G \tag{8}$$

or

$$\frac{\Delta I_{eq}}{4kT} = \frac{Wmn_i}{2qL} \int_0^\infty \mu_0^2(x) E_x(x) \frac{\partial \mu_0(x)}{\partial x} [\exp(U) - \exp(U_B)] \, dx \tag{9}$$

Assuming $\mu_0(0) = 0$ and $\mu_0(\infty) = \mu_B$, after some manipulation, we get

$$\frac{\Delta I_{eq}}{4kT} = \frac{Wmn_i}{2Lq} \overline{\{E_x(x)[\exp(U) - \exp(U_B)]\}} \frac{\mu_B^3}{3}. \tag{10}$$

Here —— denotes a suitably defined average which can be evaluated only when the exact dependence of $\mu_0(x)$ on x is known.

Alternatively, one can show,

$$\frac{\Delta I_{eq}}{4kT} = \frac{Wmn_i}{2Lq} \left(\overline{\mu_0^2(x) \frac{\partial \mu_0}{\partial x}} \right) \frac{kT}{q} \exp[U_s]. \tag{11}$$

Here —— denotes another suitably defined average. We have assumed here a sufficiently doped P type background so that U_B is a large negative number and hence $\exp[U_B] \ll 1$.

Dependence of ΔI_{eq} on gate bias

As one increases the gate bias to produce stronger inversion, the surface potential U_s increases and so do the surface electric fields. Hence the quantity

$$E_x(x) [\exp(U) - \exp(U_B)]$$

goes up. An exact evaluation is not possible in view of the absence of the knowledge of $\mu_0(x)$ profile. However, one can infer from eqn (10) that as V_G goes up, ΔI_{eq} will go up too.

Dependence of ΔI_{eq} on temperature.

In order to extract the temperature dependence we concentrate on eqn (11). Substituting $n_i = \text{const.}\ T^{3/2} \exp(qE_g/2kT)$ in eqn (11) and neglecting the temperature dependence of $\mu_0^2(x)\partial\mu_0/\partial x$ in comparison with the temperature dependence of the exponential, we get

$$\Delta I_{eq} \,\alpha\, T^{7/2} \exp\left(\frac{q(\psi_s - E_g/2)}{kT}\right) \tag{12}$$

where E_g is the band gap of the semiconductor. Now, so long as the Fermi level does not touch the conduction band the exponent is negative. Hence, a lowering of the temperature T will lower ΔI_{eq}. Both these dependences are in qualitative agreement with the experimental findings.

Note that for the degenerate case (i.e. when the Fermi level lies above the conduction band) eqn (12) does not seem to point to the right temperature dependence. However, in this range our assumption of using Boltzmann Statistics also is not valid. Quantization effects further make the situation more complicated with no simple method of formulating the problem.

REFERENCES

1. K. Takagi and A. Van der Ziel, *Solid-St. Electron.* **22**, 87 (1978).
2. R. P. Jindal and A. van der Ziel, *2nd Int. Symp. on 1/f Noise*, 17–20 March, 1980.