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Autocorrelation Function for Thermal Noise in the Case of Maser Amplifiers

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Abstract—This paper gives the autocorrelation function of thermal noise when the zero point energy noise term is neglected. Such a situation applies to maser amplifiers.

IN the discussion of noise in maser amplifiers, the zero point energy noise is always neglected [1], [2]. Van der Ziel [3] has shown from first principles that a maser does not amplify zero point energy noise and hence this neglect is justified. Therefore, we calculated the autocorrelation function of the part of the noise that is amplified; the result is presented here.

The expression for thermal noise voltage spectrum neglecting the zero point energy contribution is given by

$$S_v(f) = \frac{4hf}{e^{hf/kT} - 1} R. \tag{1}$$

Hence, the autocorrelation function is given by

$$R(s) = \overline{V(t)\, V(t+s)} = \int_0^\infty S_v(t) \cos \omega s \, df$$

$$= \int_0^\infty \frac{4hfR}{e^{hf/kT} - 1} \cos \omega s \, df. \tag{2}$$

Substituting $x = hf/kT$ and $a = kT/\hbar s$ in (2), we get

$$\overline{V(t)\, V(t+s)} = \frac{k^2 T^2}{h} 4R \int_0^\infty \frac{x}{e^x - 1} \cos (ax)\, dx. \tag{3}$$

Integrating we get

$$\overline{V(t)\, V(t+s)} = \frac{k^2 T^2}{h} 4R \sum_{n=1}^{\infty} \frac{n^2 - a^2}{(n^2 + a^2)^2} = \overline{V^2} C(s) \tag{4}$$

where

$$\overline{V^2} = \frac{k^2 T^2}{h} 4R \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{k^2 T^2}{h} 4R \frac{\pi^2}{6} \tag{5a}$$

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TABLE I
$C(s)$ AS A FUNCTION OF $|a| = |s|\, kT/\hbar$

|a|	C(s)	|a|	C(s)
0.00000	1.00000	1.80000	0.09367
0.10000	0.98057	1.90000	0.08412
0.20000	0.92573	2.00000	0.07595
0.30000	0.84460	2.10000	0.06890
0.40000	0.74884	2.20000	0.06279
0.50000	0.64939	2.30000	0.05745
0.60000	0.55448	2.40000	0.05277
0.70000	0.46905	2.50000	0.04863
0.80000	0.39516	2.60000	0.04496
0.90000	0.33296	2.70000	0.04170
1.00000	0.28147	2.80000	0.03877
1.10000	0.23923	2.90000	0.03614
1.20000	0.20470	3.00000	0.03377
1.30000	0.17646	3.10000	0.03163
1.40000	0.15327	3.20000	0.02968
1.50000	0.13413	3.30000	0.02791
1.60000	0.11822	3.40000	0.02629
1.70000	0.10490		

and

$$C(s) = \frac{6}{\pi^2} \sum_{n=1}^{\infty} \frac{n^2 - a^2}{(n^2 + a^2)^2}. \tag{5b}$$

This was directly evaluated by summing the series up to 5000 terms and applying correction for truncation. The results presented in Table I are accurate to better than 0.05 percent. The results are presented graphically in Fig. 1. We have hereby calculated the autocorrelation function for the part of thermal noise which is pertinent in case of maser.

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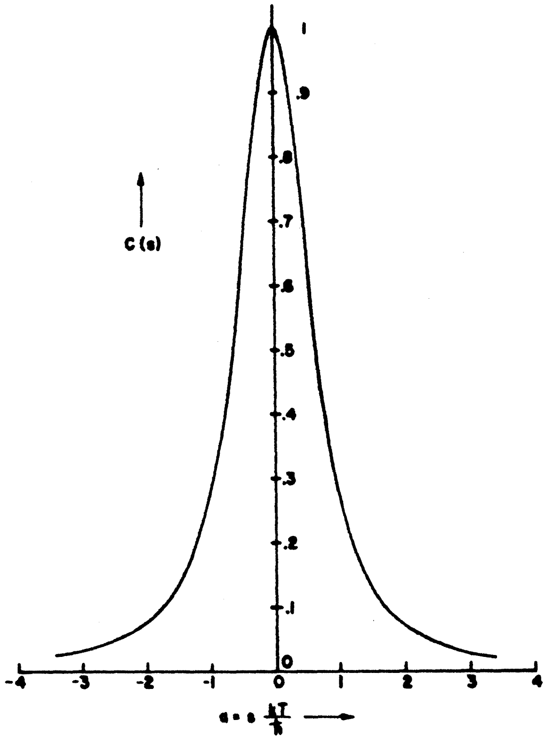

Fig. 1. $C(s)$ versus $skT/\hbar$.

REFERENCES

[1] N. Bloembergen, "Solid state masers," in *Progress in Low Temperature Physics*, vol. 3. Amsterdam, The Netherlands: North-Holland, 1961, pp. 396–429.

[2] J. P. Wittke, "Molecular amplification and generation of microwaves," *Proc. IRE*, vol. 45, pp. 291–316, 1957.

[3] A. van der Ziel, "The effect of zero point energy noise in maser amplifiers," *Physica*, vol. 96B, pp. 325–326, 1979.