A MODEL FOR 1/f MOBILITY FLUCTUATIONS IN ELEMENTAL SEMICONDUCTORS

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INTRODUCTION

$1/f$ noise in semiconductor devices has been explained using the number fluctuation model [1-3] as well as the mobility fluctuation formula [4,5]. The first experimental hint towards a connection between lattice scattering and mobility fluctuation was by Hooge and Vandamme [6]. Using the idea of phonon population fluctuations [7,8] we shall establish this connection in detail. Also, the effect of a finite electric field will be examined.

ESTABLISHMENT OF g-r SPECTRUM FOR PHONON POPULATION FLUCTUATIONS

Consider phonons with wave vector q. The number of such phonons $= n_q$. From statistical considerations, we have

$$\overline{n_q} = 1/[(\exp(\hbar\omega_q/KT) - 1]. \tag{1}$$

Also we have

$$\overline{\Delta n_q^2} = \overline{n_q}(1 + \overline{n_q}). \tag{2}$$

Both eqs (1) and (2) do not refer to any specific mechanism for this phonon population fluctuation. In general, there are a number of mechanisms which can give rise to phonon scattering and hence population fluctuations. From the experimental data [9,10], for germanium and silicon, it can be inferred that thermal conductivity, above 30K is dominated by isotope scattering for fairly pure samples. For samples with higher doping, phonon scattering from chemical impurities lowers the thermal conductivity further. However, whatever be the mechanism, the existence of time constant τ_q gives,

$$S_{\Delta n_q}(f) = \overline{\Delta n_q^2}\,\frac{4\tau_q}{1+\omega^2\tau_q^2} \tag{3}$$

ESTIMATION OF τ_q FOR ISOTOPE SCATTERING

From [9], we get

$$\tau_q = \frac{4\pi}{sq^4}\,\frac{M^2}{\overline{\Delta M^2}}\left(\frac{\rho}{m}\right) \tag{4}$$

$$m = \text{atomic mass},\ \overline{\Delta M^2} = \sum_j x_j(\Delta M_j)^2$$

where x_j is the fraction of isotopes with mass M_j and ρ = density of crystal.

Substituting typical numbers for silicon, i.e., $\overline{\Delta M^2} = 0.209$, $M = 28$, $\rho = 2.33\ \text{g/cm}^3$, $s = 8\times10^3$ m/sec, $m = 28$ atomic mass units we get

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$$\tau_q = (\lambda_{cm} \times 3.7 \times 10^4)^4 \quad \text{sec.} \tag{5}$$

Then for $\tau_q = 10^{-6}$ sec, $\lambda = 86\text{Å}$, and for $\tau_q = 10^6$ sec, $\lambda = 8.6\ \mu$m. Thus for a range of 12 decades, the corresponding phonon wave lengths are quite meaningful.

ABSOLUTE LIMITS FOR 1/f SPECTRUM

The limits of τ_q are the limits of the $1/f$ spectrum. An upper bound can be derived by taking $\lambda =$ lattice spacing.

$$\therefore \quad \text{for} \quad \lambda = 5\text{Å}, \quad f_{max} = 1.4 \times 10^{10}\ \text{Hz}$$

Also, since the $q = 0$ mode always exists (if not for the sample then for the noise cage as a whole), it would seem that there is no lower limit for the $1/f$ spectrum. However, bounds more restrictive than those derived above, might exist, from other considerations.

PHONON MEAN-FREE PATH AND SAMPLE LENGTH

Note that even for $\tau_q = 10^{-6}$ sec, the phonon has a mean-free path of 0.8 cm. It therefore seems likely that for a sample whose dimensions are smaller than 0.8 cm, boundary scattering [11] should become important.

However, one should note that this is true only for "perfectly rough" surfaces. From [12,13], it is clear, that the roughness of the surface depends upon the phonon wave length. Thus for surface roughness $<<86\text{Å}$ (corresponding to $\tau_q = 10^{-6}$ sec), we will have specular reflection which does not give rise to scattering. Hence the estimate of mean-free path ℓ_q and relaxation time τ_q are valid.

MEAN-FREE PATH FORMULATION FOR CARRIERS

In order to determine the momentum relaxation time, we shall evaluate the collision term in Boltzmann Transport Equation.

From time dependent perturbation theory, we get

$$\left(\frac{\partial f}{\partial t}\right)_{coll} = \frac{\epsilon_{ac}^2}{(2\pi)^2} \frac{1}{\rho} \int \frac{q^2}{\omega_q} \Big[(n_q+1) f(\underline{k}) \left(1 - f(\underline{k}')\right)$$

$$- n_q f(\underline{k}') \left(1 - f(\underline{k})\right)\Big] \delta\left(\epsilon(\underline{k}) - \epsilon(\underline{k}')\right) d^3k' \tag{6}$$

To be able to extract momentum relaxation time under low electric field conditions, we have to assume

$$\frac{\overline{n_q} - n_q}{\overline{n_q} + 1} < 1$$

or for $\overline{n_q} >> 1$, we have

$$\Delta n_q < \overline{n_q} \tag{7}$$

Now

$$\overline{\Delta n_q^2}\Big|_{actual} = \bar{n}_q (1 + \bar{n}_q)$$

Therefore, let

$$\overline{\Delta n_q^2}\Big|_{allowed} = \overline{\Delta n_q^2}\Big|_{actual} \alpha_a^2$$

$$= \overline{n_q}(1 + \overline{n_q}) \alpha_a^2, \quad 0 < \alpha_a < 1. \tag{8}$$

We shall refer to this as the small fluctuation approximation. Then under the small fluctuation approximation and assuming elastic collisions $[f_0(\underline{k}) = f_0(\underline{k}')]$, we get after some manipulation

$$\frac{1}{\ell} = \frac{1}{v\tau} = \frac{1}{4\pi} \int\int \frac{1}{\ell(\theta,\phi)} (1 - \cos\theta) \sin\theta \, d\theta \, d\phi \tag{9}$$

$\ell(\theta,\phi)$ = average distance travelled by carriers before they are scattered in (θ,ϕ) direction. In this case, it is independent of the electron energy.

FLUCTUATIONS IN CARRIER MEAN-FREE PATH

$$\frac{1}{\ell(\theta,\phi)} = \frac{m^2 \epsilon_{ac}^2}{\pi \hbar^4 C_1} \hbar\omega_q n_q. \tag{10}$$

Using (3) and (8), we get

$$\therefore \quad S_\ell(\theta,\phi)(f) = \overline{\ell(\theta,\phi)}^2 \frac{4\tau_q}{1+\omega^2\tau_q^2} \alpha_a^2. \tag{11}$$

We shall now *average* this over electrons travelling along the z axis with different *speeds* (Fig. 1).

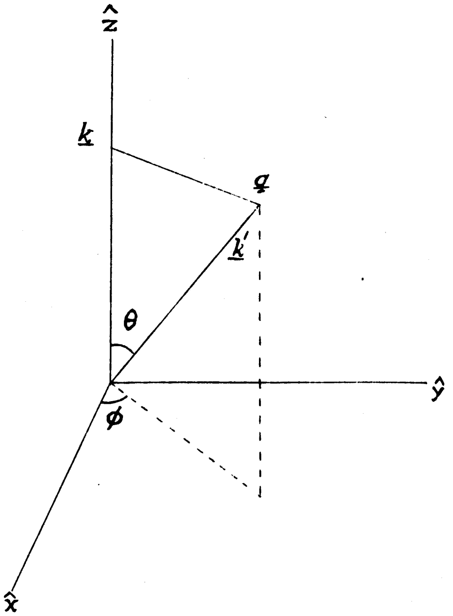

Fig. 1 Coordinate axes showing the direction of motion of the carrier along the vector $\underline{k}$ before scattering and along $\underline{k}'$ after scattering.

Now $\sin\frac{\theta}{2} = \frac{q}{2k}$.

Hence, as we vary k, keeping θ and $\hat{q}$ constant, we are in effect varying the magnitude q. However, q is related to τ. Thus averaging over energy is effectively an average over a time constant distribution. We shall presently show that,

$$g(\tau)d\tau = \frac{d\tau}{\tau} \frac{1}{\ln(\tau_1/\tau_0)}. \tag{12}$$

Now

$$\overline{\ell(\theta,\phi)} = \frac{m^2 \epsilon_{ac}^2 \hbar\omega_q}{h^4 \pi C_1} \overline{n_q} = \frac{m^2 \epsilon_{ac}^2}{\hbar^4 \pi C_1} KT. \tag{13}$$

This is independent of q. Therefore, averaging over energy and dropping the subscript q, we get from (11)

$$S_{\ell(\theta,\phi)}(f) = \frac{\overline{\ell(\theta,\phi)}^2}{ln(\tau_1/\tau_0)} \frac{\alpha_a^2}{f} \quad \text{for} \quad \frac{1}{\tau_0} << \omega << \frac{1}{\tau_1} \tag{14}$$

From (9) and (14) we get

$$\frac{S_\ell(f)}{\ell^2} = \frac{\alpha_a^2}{ln(\tau_1/\tau_0)} \frac{1}{f} . \tag{15}$$

Now $\mu = \left[\frac{e}{m^*}\right] \left\langle\frac{1}{v}\right\rangle \ell$

$$\therefore \quad \frac{S_\mu(f)}{\mu^2} = \frac{\alpha_a^2}{\ln(\tau_1/\tau_0)} \frac{1}{f} . \tag{17}$$

Since ℓ for different carriers fluctuate independently, we get for N carriers

$$\frac{S_\mu(f)}{\mu^2} = \frac{\alpha_a^2}{\ln(\tau_1/\tau_0)} \frac{1}{fN} = \frac{\alpha}{fN} \tag{18}$$

where

$$\alpha = \frac{\alpha_a^2}{\ln(\tau_1/\tau_0)} . \tag{19}$$

Note for $\frac{\tau_1}{\tau_0} = 10^{12}$ and $\alpha_a \simeq 0.2$ we get $\alpha = 1.4\times10^{-3}$ which is close to Hooge's parameter $\alpha_H = 2\times10^{-3}$. A detailed determination of α_a and its dependence on doping is still to be investigated.

DETERMINATION OF THE TIME CONSTANT DISTRIBUTION FUNCTION

The averaging over the magnitude of q is done for q pointing in a particular direction. Hence we are concerned with phonons with q pointing along a specific direction. Hence the phonon distribution can be treated to be one dimensional.

The number of states which are responsible for generating the modulation with time constant τ_q is proportional to the number of phonons with wave vector q.

Number of phonons with wave vector q

$$= \bar{n}_q = \frac{KT}{\hbar\omega_q} \quad (KT >> \hbar\omega_q) \tag{20}$$

$$\therefore \quad g(\tau)d\tau \; \alpha \bar{n}_q d_q = \frac{KT}{\hbar s} \frac{dq}{q} . \tag{21}$$

Now, for point defect scattering in general, we have

$$\tau \propto \frac{1}{q^4} \tag{22}$$

This yields,

$$\frac{dq}{q} = -\frac{1}{4} \frac{d\tau}{\tau} .$$

Comparing with (21) and normalizing, we get

$$g(\tau)d\tau = \frac{1}{\ln(\tau_1/\tau_0)} \frac{d\tau}{\tau} . \tag{23}$$

as assumed in (12).

NOISE IN PRESENCE OF OTHER SCATTERING MECHANISMS

According to the model presented here, $1/f$ noise is generated by fluctuations in the population of acoustic phonons and hence in the zero electric field acoustic mobility μ_{aco}. In the presence of other noiseless scattering mechanisms, following Hooge and Vandamme [6], we get

$$\alpha = \alpha_{ac}\left(\frac{\mu}{\mu_{aco}}\right)^2 \tag{24}$$

ELECTRIC FIELD DEPENDENCE OF 1/f NOISE

Experimentally it has been shown [16] that

$$\alpha(E) = \alpha(0)/[1 + [E/E_C')^2]. \tag{25}$$

where $\mu_0 E_C' =$ speed of sound in the medium. This can be explained [17] in terms of the present model by considering the acoustic and optical mode scattering jointly and letting only μ_{aco} generate $1/f$ noise. The field dependence for μ_{ac} that fits with the above analysis is given by

$$\mu_{ac} = \mu_{aco}[1 + (E/E_C')^2]. \tag{26}$$

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