# A General Solution for Step Junctions with Infinite Extrinsic End Regions at Equilibrium

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*Abstract*—We present here a unified solution to determine the potential profiles for step junctions in equilibrium. The complete solution is determined by solving for the two sides of the junction separately. The generalized solution consists first of a curve corresponding to majority-carrier accumulation, i.e., the low side of high-low junction. Second, it consists of a family of curves corresponding to majority-carrier depletion, i.e., the high side of a high-low junction and either side of a p-n junction. The curves in the latter case approach a common asymptote that by itself constitutes a solution for all but lightly doped sides of asymmetric p-n junctions.

Consider a general step junction with end regions of infinite length and with arbitrary values of extrinsic doping on either side of the junction. The end regions may have like-type or opposite-type doping. Let us introduce the following notation:

$U \equiv q\psi/kT$ = normalized potential;
$U_J \equiv$ normalized potential at the junction;
$U_{10} \equiv$ normalized bulk potential of side 1, to the left of the junction;
$U_{20} \equiv$ normalized bulk potential of side 2, to the right of the junction.

A well-known expression obtained by integrating what is sometimes called the Poisson–Boltzmann equation (based on equally well-known assumptions) can be written for the right-hand side of the sample as

$$\frac{x}{L_{D_i}} = \frac{1}{\sqrt{2}} \cdot \int_{U_J}^{U} \frac{dU'}{\{\cosh U' - \cosh U_{20} + (U_{20} - U')\sinh U_{20}\}^{1/2}} \tag{1}$$

where $U'$ is a dummy variable. A similar expression applies to the left of the junction. $U_J$ can be evaluated by applying the condition of field continuity at the junction, $x = 0$.

$$U_J = \frac{\cosh U_{20} - \cosh U_{10} + U_{10}\sinh U_{10} - U_{20}\sinh U_{20}}{\sinh U_{10} - \sinh U_{20}}. \tag{2}$$

Taking $U_{10}$ as a parameter, we can plot the total potential difference on the right-hand side, $U_{20} - U_J \equiv W_J$, as a function of the total potential difference in the junction-containing sample, $U_{20} - U_{10}$, as has been done in Fig. 1(a). In Fig. 1(b), we have shown a curve for a single value of $U_{10}$ to illustrate the fact that Fig. 1 applies to all step junctions, high–low as well as p-n, and have labeled various regimes. Regimes 1, 3, and 4 describe depletion of majority carriers near a junction and 2 describes the accumulation that occurs on the low side of a high–low junction.

Now let us define a new variable $W \equiv U_{20} - U$, the potential difference between the remote end region and that at an arbitrary point on the right-hand side. In terms of $W$ and $W_J$, (1) becomes, after some manipulation

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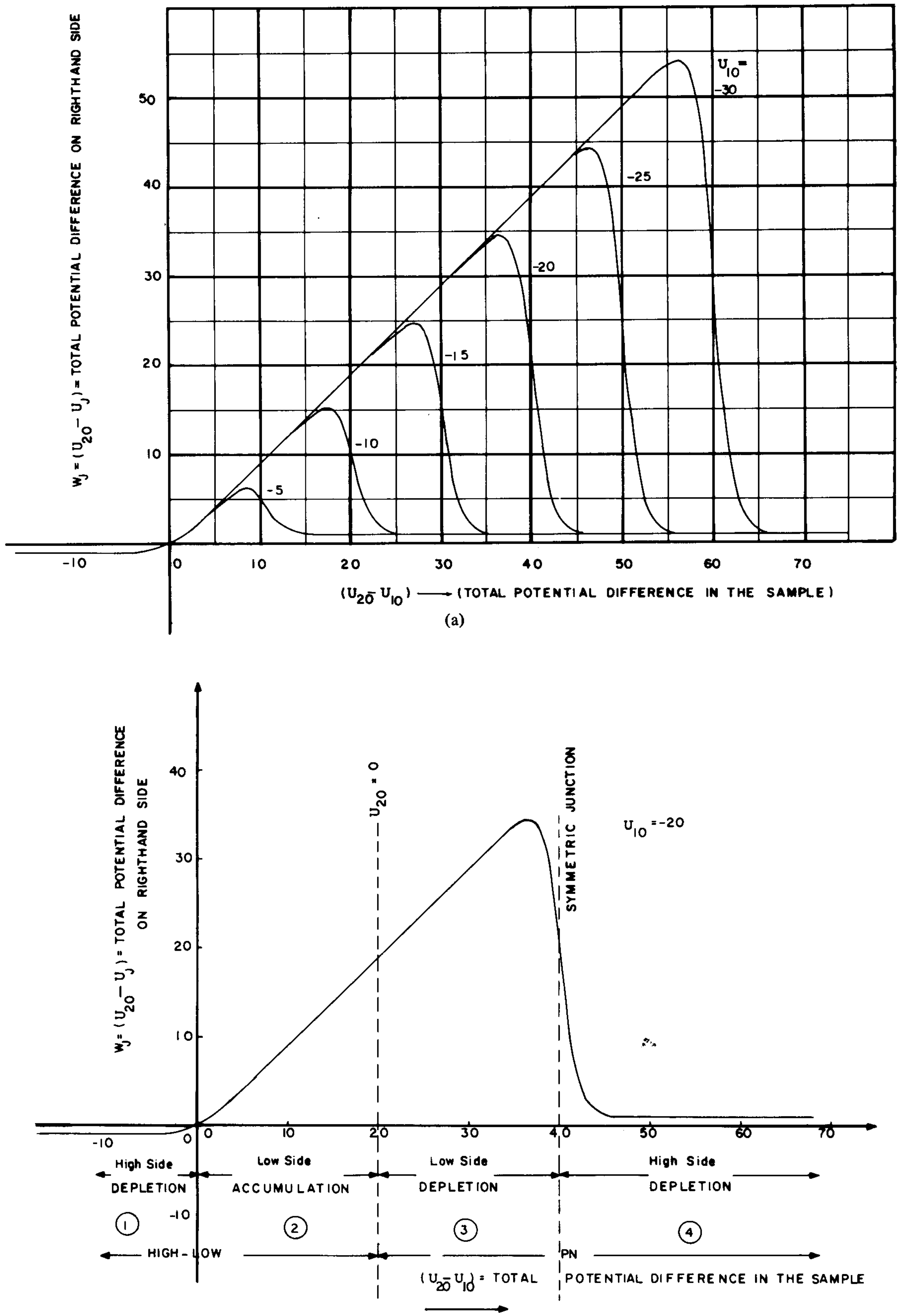


Fig. 1. (a) $(U_{20} - U_J)$ versus $(U_{20} - U_{10})$ with $U_{10}$ as a parameter. (b) $(U_{20} - U_J)$ versus $(U_{20} - U_{10})$ for $U_{10} = -20$, showing applicable regimes for high-low and p-n junctions.

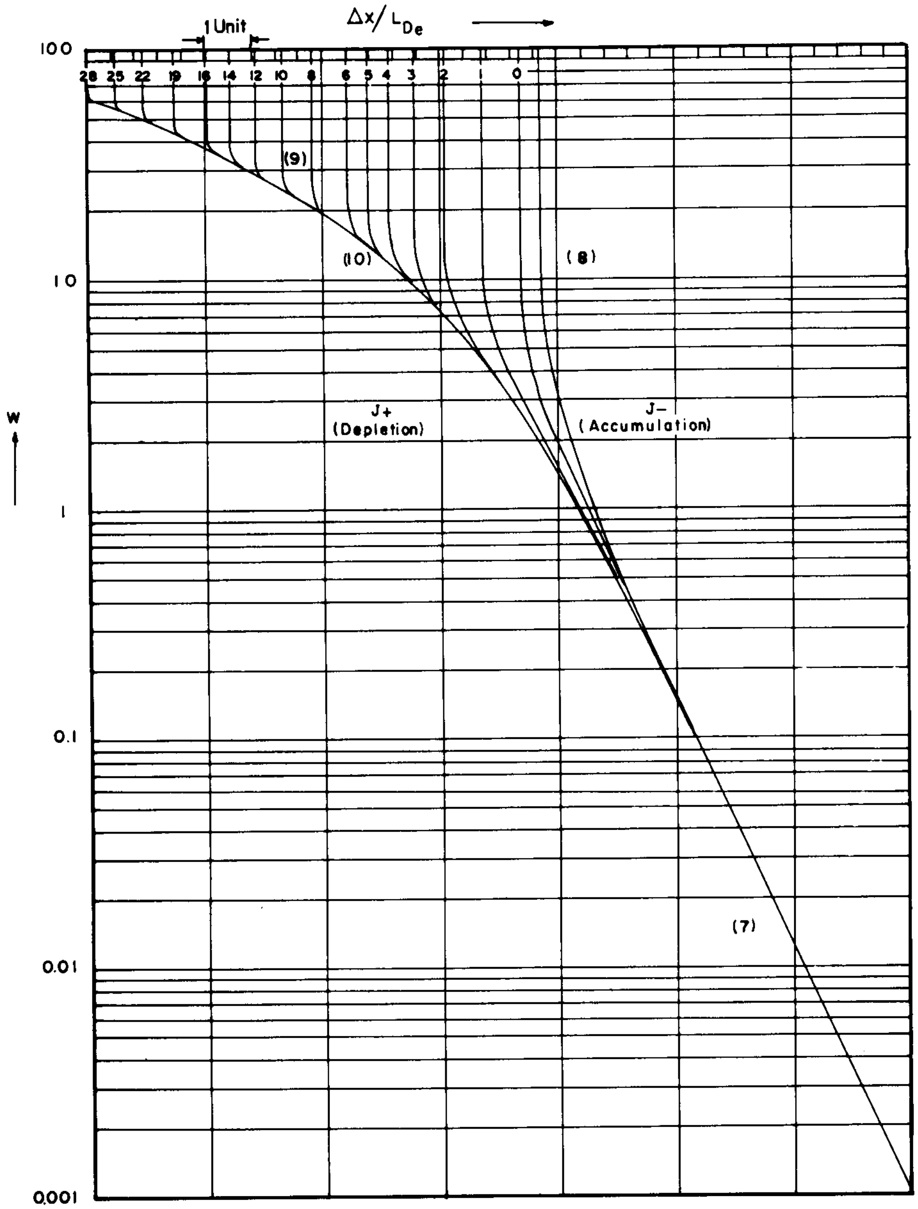


Fig. 2. $W = (U_{20} - U)$ versus normalized distance for any step junction.

$$\frac{x}{L_{De}} = \frac{1}{\sqrt{2}} \int_{W}^{W_J} \frac{dW'}{\left\{ \frac{e^{U_{20}}}{e^{U_{20}} + e^{-U_{20}}} \left(e^{-W'} - 1 + W'\right) + \frac{e^{-U_{20}}}{e^{U_{20}} + e^{-U_{20}}} \left(e^{W'} - 1 - W'\right) \right\}^{1/2}} \tag{3}$$

where we have used $L_{De} = L_{Di}(1/\sqrt{\cosh U_{20}})$. Unfortunately, this cannot be integrated analytically for all ranges of values of $W$. We will, therefore, define a new function $J_{\pm}(\alpha, \beta)$ such that for $U_{20} = \pm |U_{20}|$

$$\frac{x}{L_{De}} = J_{\pm}(\alpha, \beta) \tag{4}$$

where $\alpha$ and $\beta$ are arbitrary values of $W$ anywhere on the right-hand side of the sample, and where $\alpha$ and $\beta$ have the same sign either positive or negative. Then

$$J_{\pm}(\alpha, \beta) = \frac{1}{\sqrt{2}} \int_{\alpha}^{\beta} \frac{dW'}{\left\{ \frac{e^{U_{20}}}{e^{U_{20}} + e^{-U_{20}}} \left(e^{-W'} - 1 + W'\right) + \frac{e^{-U_{20}}}{e^{U_{20}} + e^{-U_{20}}} \left(e^{W'} - 1 - W'\right) \right\}^{1/2}}. \tag{5}$$

Now, it can be easily shown that

$$J_{\pm}(\alpha, \beta) = -J_{\mp}(-\alpha, -\beta). \tag{6}$$

Hence, there are only two linearly independent functions. Let us, therefore, concentrate only on $J_{+}(\alpha, \beta)$ where both arguments are positive numbers. Signs can be attached later on by inspection.

*Limiting Cases:* When $\alpha \ll 1, \beta \ll 1$, one can show that

$$J_{\pm}(\alpha, \beta) = (\ln \beta - \ln \alpha). \tag{7}$$

Also, for $\alpha \gg 1, \beta \gg 1$, one can show that

$$J_{-}(\alpha, \beta) = \sqrt{2}(e^{-\alpha/2} - e^{-\beta/2}) \tag{8}$$

and

$$J_{+}(\alpha, \beta) = \sqrt{2} e^{U_{20}}(e^{-\alpha/2} - e^{-\beta/2}), \qquad \beta > \alpha > |2U_{20}| \tag{9}$$

$$= \sqrt{2}(\sqrt{\beta} - \sqrt{\alpha}), \qquad \alpha < \beta < |2U_{20}|. \tag{10}$$

These functions are plotted in Fig. 2, with the four regimes identified by equation numbers.

We can think of $J_{\pm}$ as an operator operating on $\alpha$ and $\beta$ and giving the distance between the two arbitrary points on the right-hand side where $W$ reaches these values.

## Numerical Presentation of the Solution

In order to determine the solution to a given step-junction problem, one must first determine $W_J = U_{20} - U_J$. This can be done from Fig. 1(a), or from (2). It can be seen that Fig. 1(a) covers all the pertinent range of $U_{10}$ and $U_{20}$ if one remembers

$$W_J(U_{10}, U_{20}) = U_{20} - U_J = -W_J(-U_{10}, -U_{20}). \tag{11}$$

The junctions $J_{\pm}(\alpha, \beta)$ are plotted in Fig. 2. For majority-carrier accumulation (i.e., for the low side of a high–low junction), one must use $J_{-}$. For majority-carrier depletion (i.e., for high side of high–low junction or for a p-n junction), one must use $J_{+}$. It should be noted that one would have to go to the saturation region (9) of the $J_{+}$ curves only when $W_J > |2U_{20}|$. Otherwise, the asymptotic region (10) is sufficient.

Having decided on the curve to use, one picks up the values $\alpha$ and $\beta$ on the ordinate and reads off the corresponding interval $\Delta x / L_{De}$ on the abscissa.

*Typical Example:* Consider the case of a symmetric p-n junction with doping corresponding to $U_0 = +9$ on the n side of $U_0 = -9$ on the p side. Let us first solve for the p side.

Then $U_{20} = -9$, $U_{10} = +9$,

$$W_J(9, -9) = W_J(U_{10}, U_{20}) = -W_J(-U_{10}, -U_{20}) = -W_J(-9, +9).$$

From Fig. 1(a) for $U_{20} - U_{10} = 9 + 9 = 18$ and $U_{10} = -9$, we get $W_J(-9, 9) = 9$.

$$W_J(9, -9) = -W_J(-9, 9) = -9.$$

We are concerned with the magnitude of $W_J$. Since it is a p-n junction we shall use the $J_{+}$ curve. Also, it is seen that the $|U_{20}|$ branch in the $J_{+}$ family is well on the asymptotic curve for $W_J = 9$. We thus start with $W = 9$. That marks our metallurgical junction and hence the origin. From then on, values of $W$ corresponding to different distances away from the junction can be read off.

To go back to the actual normalized potential $U$, one must use $U = U_{20} - W$ and be careful about the sign, since the sign information is hidden in the transformations. However, sign can be attached by inspection since we are solving for the p side. The method is thus quite a general one, giving an answer in terms of the net doping; whether n or p is appropriate has to be decided later.

## Conclusion

We have hereby provided a general solution for all step junctions in equilibrium. An analysis relating the present results to those for the MOS case, such as the results of Kingston and Neustadter [1] and Young [2], will be presented soon.

## References


[1] R. H. Kingston and S. F. Neustadter, *J. Appl. Phys.*, vol. 26, no. 6, pp. 718–720, 1955.

[2] C. E. Young, *J. Appl. Phys.*, vol. 32, no. 3, pp. 329–332, 1961.


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