S.3:

BOLTZMANN TRANSPORT EQUATION APPROACH TO VAN DAMME'S 1/f NOISE* MODEL FOR MOSFETS

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ABSTRACT:

In a recent paper Van Damme[1] employs an independent sublayer conduction model to justify $\alpha < 2\times10^{-3}$ for 1/f noise in MOSFET inversion layer. Using the Boltzmann Transport Equation we have investigated the validity of Van Damme's assumption of independent sublayer conduction and mobility fluctuation. As a consequence, we have found that due to a finite mobility gradient, the independent sublayer conduction model fails. In order to fix it, we propose a correction factor to the local mobility, this factor being proportional to the transverse electric field due to the gate bias. However, this simple correction factor and hence even the corrected version of Van Damme's model has validity over a limited range only. An independent treatment based on a viscous force approach produces identical results, hence reenforcing the Boltzmann Transport Equation treatment of the problem.

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INTRODUCTION:

In a recent paper, Van Damme has offered an explanation for 1/f noise in a MOSFET inversion layer based on Hooge's empirical bulk effect formula. He has tried to justify the low value of α on the basis of an independent sublayer conduction model. Each sublayer has its own mobility, carrier density and noise parameter α. Total noise is calculated by assuming no correlation between noise in each sublayer and hence summing over the contribution due to each sublayer.

PROOF OF THE INCONSISTENCY:

To be specific, we shall treat the case of an n-channel enhancement mode MOSFET. Referring to Fig. 1, we know that the equilibrium distribution function in momentum space is given by

$$f_o = [\exp\{\frac{\frac{1}{2} mv_x^2 + \frac{1}{2} mv_y^2 + \frac{1}{2} mv_z^2 - qV(x) - E_F}{kT}\} + 1]^{-1} \qquad (1)$$

In the presence of a drain bias and hence an electric field E_y according to Van Damme's picture the new steady state distribution function is

$$f = [\exp\{\frac{\frac{1}{2} mv_x^2 + \frac{1}{2} m(v_y + \frac{qE_y\tau(x)}{m})^2 + \frac{1}{2} mv_z^2 - qV(x) - E_F}{kT}\} + 1]^{-1} \qquad (2)$$

i.e., we have added a drift velocity $\frac{-qE_y\tau(x)}{m}$ for each sublayer. It is easily shown that this leads to Van Damme's model.

From Boltzmann Transport Equation, in steady state, we get

$$\vec{v} \cdot \nabla_r f + \frac{\vec{F}}{m} \cdot \nabla_v f = - \frac{f(r)-f_o(r)}{\tau(x)} \qquad (3)$$

In our case f_o is given by equation (1) and f by equation (2). The force $F = -q[E_x + E_y]$. Assuming all derivations with respect to z vanish and also that $E_z = 0$ and $\frac{\partial f}{\partial y} = 0$, we get

$$\frac{-q}{m} E_x \frac{\partial f}{\partial v_x} - \frac{q}{m} E_y \frac{\partial f}{\partial v_y} + v_x \frac{\partial f}{\partial x} = - \frac{f(r) - f_o(r)}{\tau(x)} \tag{4}$$

Substituting for the required expressions and neglecting second order terms in E_y, equation (4) reduces to

$$\frac{q}{kT}[-E_y v_y f(1-f) v_x \frac{\partial \tau}{\partial x}] + E_y f(1-f) \frac{q}{kT} v_y = E_y f(1-f) \frac{q}{kT} v_y \tag{5}$$

Hence the equation does not balance with the extra term being

$$\frac{-q}{kT} E_y v_y f(1-f) v_x \frac{\partial \tau}{\partial x} .$$

This proves that the model of dividing the inversion layer into sublayers conducting independently with their respective time constants $\tau(x)$ (and carrier mobility $\mu(x) = \frac{q\tau(x)}{m}$) does not satisfy Boltzmann Transport Equation.

This further points out that if we do want to talk about sublayer conduction we will have to take into account the effect of other sublayers on the time constant and hence the mobility of a given sublayer.

As a further step we can conclude that any fluctuation in the mobility of a given sublayer is going to produce a fully correlated fluctuation component in the mobility of other sublayers. Hence, if we want to explain 1/f noise due to mobility fluctuations we will have to deal with this fully correlated noise component as well.

Derivation of Correction Factor

In the expression for the steady state distribution function f (equation 2) let us replace $\frac{qE_y\tau(x)}{m}$ by $\frac{qE_y\tau(x)}{m} \cdot C(x,v_x)$ where $C(x,v_x)$ is a dimensionless function.

Then

$$f = \left[\exp\left\{\frac{\frac{1}{2}mv_x^2 + \frac{1}{2}m\left(v_y + \frac{qE_y\tau(x)}{m}C(x,v_x)\right)^2 + \frac{1}{2}mv_z^2 - qV(x) - E_F}{kT}\right\} + 1\right]^{-1} \qquad (6)$$

Substituting (6) in the Boltzmann equation (4) we find that $C(x,v_x)$ obeys the following differential equation

$$C(x,v_x) = 1 - v_x\frac{\partial(C\tau)}{\partial x} + \frac{qE_x\tau}{m}\cdot\frac{\partial C}{\partial v_x} \qquad (7)$$

Using an iterative technique to solve this, we obtain a series solution for the correction factor C. Taking average over all velocities and neglecting higher order terms, we get

$$C = 1 - \frac{qE_x\tau}{m}\cdot\frac{\partial\tau}{\partial x}$$

Alternative derivation of the correction factor:

Having derived an expression for C from momentum space considerations, we shall now proceed to derive the same result from real space, kinetic theory considerations. We assume that the whole inversion layer can be divided into sublayers characterized by a mobility μ and a carrier density n. Then, when we impress an electric field E_y due to an application of the drain bias, there will be a drift set up in each sublayer. This drift velocity is simply given by

$$v_d = \frac{-qE_y \tau(x)}{m}$$

Hence we see that each sublayer has a different drift momentum. This is analogous to the situation of laminer viscous flow of a fluid by the application of an external force. Since, each sublayer is constantly exchanging carriers with its neighbors they mutually exchange this drift momentum. This net flow of momentum per unit time results in a mutual force between two sublayers which is usually called the viscous force.

On the basis of detailed kinetic theory considerations in the presence of a finite ∇n and $\nabla \mu$, it can be shown that the net force exerted per unit area on a sublayer of thickness Δx and centered at the origin is given by

$$= \frac{1}{3} n \overline{v^2} \tau \left.\frac{dv_d}{dx}\right|_{x = \frac{\Delta x}{2}} - \frac{1}{3} n \overline{v^2} \tau \left.\frac{dv_d}{dx}\right|_{x = -\frac{\Delta x}{2}}$$

Number of particles experiencing this force = $n(o) \cdot \Delta x \cdot 1$.

In the limit $\Delta x \to 0$, this gives average force per particle

$= \frac{1}{3n}(-qE_y)\frac{\partial}{\partial x}(n \overline{v^2} \tau \frac{\partial \tau}{\partial x})$. After detailed evaluations and neglecting higher order terms, we get once more

$$C = 1 - \frac{qE_x \tau}{m} \cdot \frac{\partial \tau}{\partial x}$$

It should be emphasized that this type of simple correction factor only holds if the correction factor is close to unity. If this is not the case, the independent sublayer conduction model cannot be fixed at all.

Hence, under these restrictions, the current density in a sublayer is given by:

$$J_y = \frac{nq^2\tau(x)}{m} E_y\{1 - \frac{qE_x\tau}{m} \frac{\partial\tau}{\partial x}\}$$

Denoting $\mu_o(x) = \frac{q\tau(x)}{m}$, we get

$$J_y = nq\mu_{eff}E_y \quad \text{where} \quad \mu_{eff} = \mu_o\{1 - \frac{qE_x\tau}{m} \frac{\partial\tau}{\partial x}\}$$

Let $v_{dx} = \frac{q\tau(x)}{m} E_x$

Then $\mu_{eff} = \mu_o(x)\{1 - v_{dx} \frac{m}{q} \frac{\partial\mu_o}{\partial x}\}$

CONCLUSION:

We have shown that Van Damme's independent sublayer conduction model is inconsistent with Boltzmann Transport Equation. It can, however, be fixed for a limited range by introducing a simple correction factor. In order to estimate the contribution of mobility fluctuations to 1/f noise, one would have to take into account both the correlated as well as uncorrelated components of noise due to each sublayer.

Reference

1. Model for 1/f Noise in MOS Transistors Biased in the Linear Region, L. K. J. Van Damme, to be published, S.S.E. 1980.

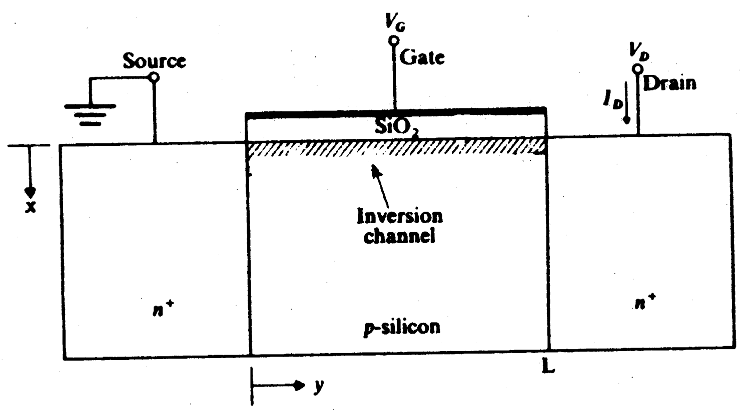

FIGURE-1
Idealized *n*-channel MOSFET.