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GENERATION-RECOMBINATION NOISE AT 77°K IN SILICON BARS AND JFETs†

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Abstract—The theory of generation-recombination noise in silicon bars and in JFETs is extended to the case in which the devices operate in the hot electron regime. It is shown that Nougier *et al.*'s measurements at 77°K can at least be partly explained as generation recombination noise; as a matter of fact, the theory can provide an almost perfect match for field strengths between 1000 and 3000 V/cm for g–r noise alone. We believe, however, that some hot electron noise with a similar field dependence as g–r noise is present. One can now understand why Kim, van der Ziel and Rucker found an activation energy of only 63 mV for the noise in silicon JFETs around 77°K.

In a recent paper Nougier *et al.*[1] measured noise in silicon bars and silicon n-channel JFETs at 77°K and interpreted the noise as hot electron noise. Kim, van der Ziel and Rucker[2] measured the same noise in n-channel JFETs in a temperature range near 77°K and found that the noise resistance R_n had an activation energy of 0.063 eV; they, therefore, interpreted the noise as generation-recombination (g–r) noise, for which one would expect an activation energy of that order of magnitude. Moreover, Kim, van der Ziel and Rucker[3] measured hot electron noise in silicon JFETs between 150 and 300°K; when their measurements are extrapolated to 77°K the expected hot electron noise is about an order of magnitude less than the noise that is actually observed. This means that the noise observed in silicon devices at 77°K is most likely not all hot electron noise, but that there must be some g–r noise present.

It is the aim of this paper, therefore, to extend the theory of g–r noise to the hot electron regime, both for silicon bars and for n-channel silicon JFETs.

We first turn to the case of silicon bars. If V is the applied voltage and $G = q\mu(E)N/L^2$ the d.c. conductance, where q is the electron charge, $\mu(E)$ the field-dependent mobility, N the number of carriers in the sample, and L the device length, then the current is given as

$$I = GV = q\mu_n(E)NV/L^2. \quad (1)$$

Generation-recombination noise is caused by fluctuations δN in N, so the fluctuating short-circuited noise current is

$$\delta I = [q\mu_n(E)V/L^2]\delta N,$$

or

$$S_I(f) = [q\mu_n(E)V/L^2]^2 S_{\delta N}(f) \quad (2)$$

where $S_I(f)$ and $S_{\delta N}(f)$ are the respective spectra. But we know that[4]

$$S_{\delta N}(f) = \overline{4\delta N^2}\tau/(1+\omega^2\tau^2) \quad (3)$$

where τ is the lifetime of the carriers. Putting $\overline{\delta N^2} = \alpha N$, and substituting into (2) yields

$$[S_I(f)]_{g-r} = 4E^2 q\mu_n(E)G\alpha\tau/(1+\omega^2\tau^2) \quad (4)$$

where $E = V/L$ is the field strength and α and τ can be evaluated with the help of generation-recombination statistics[2]. We shall see that both α and τ decrease with increasing field strength E.

According to Nougier *et al.*[1] the noise temperature of the electrons in the presence of hot electron effects follows from

$$[S_I(f)]_n = 4kT_e dI/dV = 4kT_e\mu_n'(E)qN/L^2 \quad (5)$$

where $\mu_n'(E) = \mu_n(E) + E\,d\mu_n(E)/dE$ is the differential mobility of the carriers. Equating (5) and (4), one obtains for the apparent electron temperature $(T_e)_{g-r}$ due to g–r noise at low frequencies ($\omega^2\tau^2 \ll 1$).

$$(T_e)_{g-r} = (qE^2/k)\alpha\tau\mu_n(0)\cdot\mu_n^2(E)/[\mu_n(0)\mu_n'(E)]. \quad (6)$$

Before comparing this with experiment, we must first evaluate α and τ as functions of the field strength E. We start from the generation and recombination rates $g(n)$ and $r(n)$, respectively

$$g(n) = \gamma(N_d - n) \quad r(n) = \rho n^2 \quad (7)$$

where γ and ρ are constants, N_d is the donor concentration and n the electron concentration. Here ρ is independent of the field E, but γ increases with increasing field due to the Poole–Frenkel effect, which is the Schottky effect for donors in a high field[5]. According to this effect the field decreases the binding energy of the electrons to the donors from E_0, the value without field, to the value $E_0 - \Delta V$ with field, where

$$\Delta V = (qE)^{1/2}/(\pi\epsilon\epsilon_0)^{1/2} = 7.58\times 10^{-5}(E/\epsilon)^{1/2} \quad (8a)$$

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where ϵ is the relative dielectric constant, $\epsilon_0 = 8.85 \times 10^{-12}$ F/m, and E is in V/m, so that

$$\gamma = \gamma_0 \exp(q\Delta V/kT). \tag{8b}$$

In equilibrium $g(n) = r(n)$ and $n = n_0$. Introducing $u = n_0/N_d$ as the value with field and u_0 as the corresponding value without field, we have

$$\frac{u^2}{1-u} = \frac{\gamma_0}{\rho N_d} \exp\left(\frac{q\Delta V}{kT}\right); \quad \frac{u_0^2}{1-u_0} = \frac{\gamma_0}{\rho N_d}. \tag{9}$$

Furthermore, the lifetime τ of the carriers *with field* follows from[4]

$$\frac{1}{\tau} = \left[-\frac{dg(n)}{dn} + \frac{dr(n)}{dn}\right]_{n_0} = \gamma \frac{2N_d - n_0}{n_0}$$

$$= \gamma_0 \frac{2-u}{u} \exp\left(\frac{q\Delta V}{kT}\right) \tag{10a}$$

whereas the lifetime τ_0 of carriers *without field* follows from

$$\frac{1}{\tau_0} = \gamma_0 \frac{2-u_0}{u_0}. \tag{10b}$$

Table 1 gives the measured values of $\mu_n(E)/\mu_n(0)$, $\mu_n'(E)/\mu_n(0)$ and T_e as functions of the field strength E[1]. Also shown is the value of $\alpha\tau$ that provides a perfect match to the data.

Table 1. $\mu_n(E)/\mu_n(0)$, $\mu_n'(E)/\mu_n(0)$ and T_e in °K as functions of E (6 Ωcm material)

E (in V/cm)	$\mu_n(E)/\mu_n(0)$	$\mu_n'(E)/\mu_n(0)$	T_e (in °K)	$\alpha\tau$ in s
1000	0.56	0.31	1700	1.035×10^{-11}
2000	0.38	0.155	3500	0.578×10^{-11}
3000	0.30	0.096	5000	0.365×10^{-11}

The definition of α is[2]

$$\alpha = \frac{1-u}{2-u}; \quad \alpha_0 = \frac{1-u_0}{2-u_0}. \tag{11}$$

According to Nash and Holm-Kennedy we have at 77°K that $\mu_n(0) = 1.4 \times 10^4$ cm^2/Vs, $u_0 = 0.681$ and hence $\alpha_0 = 0.242$, so that $\gamma_0/(\rho N_d) = 1.454$, for 6 Ωcm material[6]. Since silicon has $\epsilon = 12$, we have the following table for ΔV.

Table 2. ΔV as a function of the field strength E

E (in V/cm)	ΔV in mV
1000	6.92
2000	9.79
3000	11.99

†The Klaassen–Prins method is valid here, since $G_u(x)\Delta V(x, t) \equiv 0$, at $x = 0$ and $x = L$, when the device is h.f. short-circuited. Moreover, van Vliet (unpublished) applied the impedance field method to the JFET g–r noise and obtained perfect agreement with the Klaassen–Prins method at low fields.

Furthermore, we have the following tabel for α/α_0, τ/τ_0 and the product.

Table 3. α/α_0, τ/τ_0 and $\alpha\tau/(\alpha_0\tau_0)$ as functions of E

E (in V/cm)	α/α_0	τ/τ_0	$\alpha\tau/(\alpha_0\tau_0)$	$\alpha_0\tau_0$ (in s)
1000	0.597	0.485	0.290	3.58×10^{-11}
2000	0.448	0.344	0.154	3.75×10^{-11}
3000	0.352	0.268	0.0943	3.87×10^{-11}

From the values of $\alpha\tau$ found in Table 1 we have also evaluated the value of $\alpha_0\tau_0$ that provides a perfect match at each field. Its average value is 3.73×10^{-11} s and, since $\alpha_0 = 0.242$, we find $\tau_0 = 1.54 \times 10^{-10}$ s. Since the deviation from the mean is relatively small, this is a reasonably accurate value. We thus obtain the following end result (Table 4). The agreement is excellent.

Table 4. Experimental and theoretically matched values of T_e

E (in V/cm)	$(T_e)_{exp}$ in °K	$(T_e)_{theor.}$ in °K
1000	1700	1800
2000	3500	3500
3000	5000	4800

This means that our theory could fully explain the observations of Nougier *et al.*[1] as being caused by generation-recombination noise only. But since our approach does not distinguish between hot electron noise and g–r noise when the two have the same field dependence, it is safer to assume that both processes play a part.

We finally turn to the generation-recombination noise in JFETs. In evaluating the noise the Klaassen–Prins† method will be used[7]. Starting from the Langevin equation

$$\Delta I(t) = \partial[G_u(x)\Delta V(x, t)]/\partial x + H(x, t) \tag{12}$$

where $H(x, t)$ is a random distributed noise source, $G_u(x) = q\mu_n(E)nA$ the d.c. conductance for unit length and $\Delta V(x)$ the resulting noise voltage at x: here n is the carrier density, A the cross section area of the channel at x, and $\Delta I(t)$ the resulting noise current in the external circuit. If source and drain are h.f. connected, one obtains by integrating over the device length L

$$\Delta I(t) = \frac{1}{L}\int_0^L H(x, t)\partial x$$

or

$$S_I(f) = \frac{1}{L^2}\int_0^L \int_0^L S_H(x, x', f)\,dx\,dx' \tag{13}$$

where $S_H(x, x', f)$ is the cross-correlation spectrum of $H(x, t)$. In the first integral the time t must be kept constant in the integration process.

To find $S_H(x, x', f)$ for this case we replace in eqn (4) G by $G_u(x')$, E^2 by $E(x)E(x')$, multiply by the δ-function

$\delta(x'-x)$ and obtain

$$S_H(x, x', f) = 4E(x)E(x')q\mu_n(E)G_u(x')\delta(x'-x)\,\alpha\tau/(1+\omega^2\tau^2). \qquad (14)$$

Bearing in mind that $I = -G_u(x)E(x)$ is the d.c. current and that $u_d(E) = -\mu(E)E(x)$ is the drift velocity at x, yields

$$S_I(f) = \frac{4qI}{L^2}\int_0^L u_d(E)\,\mathrm{d}x\,\frac{\alpha\tau}{1+\omega^2\tau^2} \qquad (15)$$

which is the extension of the low-field g–r noise formula[8] into the hot electron regime. In the low-field case the integral has the value $\alpha_0\tau_0\mu_n(0)V_d/(1+\omega^2\tau_0^2)$, where V_d is the drain voltage; this gives agreement with the previous theory. In the high-field case the integral must be evaluated numerically.

Since at low fields $S_I(f)$ should have an activation energy, determined by $\alpha_0\tau_0$, of about $2E_0$ (or about 0.088 V), the activation energy with field should be determined by the weighted average $\overline{\alpha\tau}$, which would give $2(E_0-\overline{\Delta V})$. Since Kim *et al.* measured an activation energy of 0.063 V, this corresponds to $\overline{\Delta V} = 12.5$ mV. In view of the previous discussion this is not unreasonable.

All data are, therefore, compatible with the idea that both the hot electron noise and the g–r noise contribute to the measured electron temperature T_e. If $(T_e)_h$ and $(T_e)_{g-r}$ are the contributions of the hot electrons and of the g–r processes, respectively, we have

$$T_e = (T_e)_h + (T_e)_{g-r}. \qquad (16)$$

The measurements by Kim *et al.*[2, 3] seem to indicate that the contribution of $(T_e)_{g-r}$ is quite significant.

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