Computational Formulas for ANOVA

One-Way ANOVA

Let $a = \# \text{ of levels of the independent variable} = \# \text{ of groups}$

$N = \text{total \# of observations in the experiment}$

$n_1 = \# \text{ of observations in group 1, etc.}$

$H_0$: $\mu_1 = \mu_2 = \mu_3 = \ldots = \mu_a$

ANOVA analyzes sample variances to draw inferences about population means. Sample variances can always be calculated as $SS/df$ and these sample variances are called mean squares ($MS$):

$$SS_{Total} = \sum X^2 - \left(\frac{\sum X}{N}\right)^2$$

$$df_{Total} = N - 1$$

$$SS_{Between} = \sum_{i=1}^{a} \frac{\sum X_i^2}{n_i} - \frac{\sum X^2}{N}$$

$$df_{Between} = a - 1$$

$$SS_{Within} = SS_{Total} - SS_{Between}$$

$$df_{Within} = N - a$$

$$s^2 = \frac{SS}{df} = MS$$

$$F = \frac{MS_{Between}}{MS_{Within}}$$

Example.

<table>
<thead>
<tr>
<th></th>
<th>$X_1$</th>
<th>$X_2$</th>
<th>$X_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Placebo</td>
<td>9</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>8</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>Sum</td>
<td>40</td>
<td>25</td>
<td>15</td>
</tr>
<tr>
<td>$M$</td>
<td>8</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>$s$</td>
<td>1.224745</td>
<td>1.870829</td>
<td>1.581139</td>
</tr>
</tbody>
</table>

ANOVA Summary Table

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between</td>
<td>63.333</td>
<td>2</td>
<td>31.667</td>
<td>12.67</td>
<td>0.0011</td>
</tr>
<tr>
<td>Within</td>
<td>30.000</td>
<td>12</td>
<td>2.500</td>
<td></td>
<td>2.500</td>
</tr>
<tr>
<td>Total</td>
<td>93.333</td>
<td>14</td>
<td>6.667</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

SS$_T = 9^2 + 8^2 + \ldots + 1^2 + 5^2 - (80)^2/15 = 93.333$

SS$_B = (40^2 + 25^2 + 15^2)/5 - 80^2/15 = 63.333$

SS$_W = 93.333 - 63.333 = 30.000$

An alternative computational approach emphasizing the conceptual basis of ANOVA is given below.

This is the variance of all scores in the experiment = 6.667.

This is the average of the variances within the groups = 2.50.

$(1.22^2 + 1.87^2 + 1.58^2)/3 = 2.50.$

This is $n$ times the variance of the means = $5(6.333) = 31.667.$

Multiple Comparisons:

$$LSD = t_{Crit} \sqrt{\frac{2MS_{Error}}{n}}$$

$t_{Crit}$ is the critical value from a $t$-table using the $df$ of the error term from the ANOVA table. The error term is always the denominator of the $F$-ratio. Thus, in the above example, the error $df$ would be 12. The $MS_{Error}$ would be 2.50; $n$ is always the number of observations each mean you’re comparing is based on.
Two-Way Factorial ANOVA

Let:
- \( a \) = # of levels of the independent variable A
- \( c \) = # of levels of the independent variable C
- \( ac \) = # of cells in the experiment
- \( N \) = total # of observations in the experiment
- \( n_1 \) = # of observations in cell 1, etc.

\[
SS_{Total} = \sum X^2 - \left( \frac{\sum X}{N} \right)^2
\]

\[
SS_{Within} = SS_{Total} - SS_{Between}
\]

\[
SS_A = \sum (\sum \text{for each row})^2 - \left( \frac{\sum X}{n \text{ for each row}} \right)^2
\]

\[
SS_C = \sum (\sum \text{for each column})^2 - \left( \frac{\sum X}{n \text{ for each column}} \right)^2
\]

\[
SS_{AC} = SS_{Between} - SS_A - SS_C
\]