

PSYCHOLOGY 513 - QUANTITATIVE MODELS

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Assignment No. 5

1. Use the matrix commands in a spreadsheet (e.g., Excel) on the following matrices to demonstrate the fundamental verities about matrices listed below:

$$\mathbf{A} = \begin{matrix} 4 & 5 & 1 & 2 \\ 3 & 3 & 7 & 2 \\ 5 & 0 & 6 & 4 \\ 3 & 4 & 5 & 8 \\ 9 & 4 & 2 & 5 \end{matrix} \quad \mathbf{B} = \begin{matrix} 2 & 3 & 0 & 1 \\ 5 & 5 & 1 & 1 \\ 9 & 4 & 0 & 2 \\ 0 & 4 & 8 & 5 \end{matrix} \quad \mathbf{D} = \begin{matrix} 5 & 1 & 6 & 9 & 5 \\ 0 & 5 & 6 & 6 & 1 \\ 1 & 1 & 5 & 0 & 9 \\ 7 & 5 & 6 & 3 & 3 \end{matrix}$$
$$\mathbf{E} = \begin{matrix} 5 & 8 & 8 & 3 \\ 1 & 6 & 2 & 7 \\ 7 & 9 & 5 & 4 \\ 3 & 3 & 6 & 7 \end{matrix}$$

- Show that \mathbf{AD} is not equal to \mathbf{DA}
- Show that $(\mathbf{AB})' = \mathbf{B}'\mathbf{A}'$
- Find $\mathbf{A}'\mathbf{A}$ and \mathbf{AA}' and show that while they are not equal, they are both symmetric and their traces are equal. (The trace of a square matrix is simply the sum of its diagonal elements.)
- Show that $\mathbf{B}(\mathbf{B}^{-1}) = \mathbf{I}$
- Show that $(\mathbf{EB})^{-1} = (\mathbf{B}^{-1})(\mathbf{E}^{-1})$
- Show this important result: $(\mathbf{B}')^{-1} = (\mathbf{B}^{-1})'$
- Find the matrix \mathbf{C} such that $\mathbf{AB} = \mathbf{C}$. Matrix algebra allows us to solve for \mathbf{A} in the following way:

$$\begin{aligned} \mathbf{AB} &= \mathbf{C} \\ \mathbf{AB}(\mathbf{B}^{-1}) &= \mathbf{C}(\mathbf{B}^{-1}) \\ \mathbf{AI} &= \mathbf{C}(\mathbf{B}^{-1}) \\ \mathbf{A} &= \mathbf{C}(\mathbf{B}^{-1}) \end{aligned}$$

Show that, indeed, $\mathbf{A} = \mathbf{C}(\mathbf{B}^{-1})$

2. Using the police/robberies data from an earlier homework, find the values of b_0 and b_1 , i.e., the elements of the vector $\mathbf{b}' = [b_0 \ b_1]$, for the regression equation. Use the matrix formulation

$$\mathbf{b} = ((\mathbf{X}'\mathbf{X})^{-1})\mathbf{X}'\mathbf{Y}$$

with \mathbf{X} defined in an appropriate manner and the spreadsheet matrix commands to solve for \mathbf{b} . You should, of course, get the same answer you got when you did the homework originally.