CONTRASTS IN ANOVA

Generally if we have a hypothesized *pattern* of means that we are interested in detecting, we will be able to do much better than simply testing the omnibus H_0: $\mu_1 = \mu_2 = \mu_3 = ...$, which takes no account of the pattern we are interested in.

An improved hypothesis test can be carried out by testing a *contrast* or *comparison* among the means. [We use these 2 terms interchangeably.] Contrasts are constructed by specifying a set of weights [i.e., c_i; $i = 1,2, ... a$] (one for each group mean) which sum to zero [i.e., $\Sigma c_i = 0$]. The pattern of the weights (called contrast coefficients) should reflect the hypothesized pattern of interest in the means.

Various patterns among means may be of interest in other experiments, and *it is almost always better in real experiments to test precise hypotheses specified by contrasts, rather than omnibus hypotheses.*

The null hypothesis tested by a contrast may be written as follows:

$$H_0\text{: } \psi = \sum c_i \mu_i = 0$$

This fact suggests that other interesting hypotheses may be tested with contrasts. Consider an experiment with 2 experimental groups and 1 control group. If one wished to test the hypothesis that the mean for the control group was equal to the average of the means for the 2 experimental groups, one could write that hypothesis as follows:

$$H_0\text{: } \mu_C = \frac{\mu_{E_1} + \mu_{E_2}}{2} \quad \text{or, equivalently,} \quad \psi = 2\mu_C - \mu_{E_1} - \mu_{E_2} = 0$$

yielding

	C	E_1	E_2	
c_i:	2	-1	-1	for contrast weights.

Another meaningful contrast here would be to compare the 2 experimental groups.

$$H_0\text{: } \mu_{E_1} = \mu_{E_2} \quad \text{or} \quad \mu_{E_1} - \mu_{E_2} = 0 \text{, yielding}$$

	C	E_1	E_2	
c_i:	0	1	-1	for contrast coefficients.

Example 2. Test the 2 hypotheses above in an experiment with the following means and $n = 4$:

	A		
	C	E_1	E_2
	9	12	6
	4	6	8
	6	8	12
	5	10	14
Total	24	36	40
Mean	6	9	10

	A		
	C	E_1	E_2
	6	9	10
c_{1i}:	2	-1	-1
c_{2i}:	0	1	-1

The ANOVA summary table would include the following:

Source	*SS*	*df*	*MS*	*F*	*p*
A	34.667	2	17.333	2.11	.1775
ψ_1	32.667	1	32.667	3.97	.0774
ψ_2	2.000	1	2.000	0.24	.6337
S(A)	74.000	9	8.222		

The JMP analysis that would produce these results is below (Note that the above *F*s for the contrasts are equal to t^2s from the JMP analysis):

Response Y
Summary of Fit

RSquare	0.319018
RSquare Adj	0.167689
Root Mean Square Error	2.867442
Mean of Response	8.333333
Observations (or Sum Wgts)	12

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	2	34.66667	17.3333	2.1081
Error	9	74.00000	8.2222	Prob > F
C. Total	11	108.66667		0.1775

Effect Tests

Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F
A	2	2	34.666667	2.1081	0.1775

Effect Details
A
Least Squares Means Table

Level	Least Sq Mean	Std Error	Mean
C	6.000000	1.4337209	6.0000
E1	9.000000	1.4337209	9.0000
E2	10.000000	1.4337209	10.0000

Contrast
Test Detail

C	1	0
E1	-0.5	1
E2	-0.5	-1
Estimate	-3.5	-1
Std Error	1.7559	2.0276
t Ratio	-1.993	-0.493
Prob>|t|	0.0774	0.6337
SS	32.667	2

Sum of Squares	34.666666667
Numerator DF	2
Denominator DF	9
F Ratio	2.1081081081
Prob > F	0.1774635581