[Søren Eilers - Lecture 5] May 15, 2015

Plan: ① Characterize $\sim_m$ } in the unital case
② Additional moves
③ Classification up to $*$-iso
④ Extensions and phantoms
⑤ Cuntz splice invariance
⑥ Working conjecture ← for graphs w. infinite vertices
⑦ Infinite moves
⑧ Bentmann / Meyer invariant
⑨ Semiprojectivity
⑩ Naturally occurring $C^*(E)$

① E, F - irreducible, finite, essential, not a single cycle (i.e. not a permutation matrix)

$$\begin{tikzcd} C^*(E) \arrow[r, phantom, "\cong"] & C^*(F) \\ \mathcal{D}_E \otimes c_0 \arrow[u, hook] \arrow[r, phantom, "\cong"] & \mathcal{D}_F \otimes c_0 \arrow[u, hook] \end{tikzcd}$$

$$\Updownarrow$$

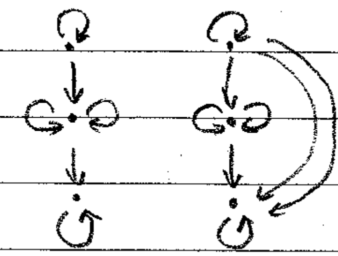
$$E \sim_m F$$

if $C^*(E)$ and $C^*(F)$ have units and are gauge simple, this remains true

what remains is the case $E^0_{sing} \neq \emptyset$, $F^0_{sing} \neq \emptyset$.

(in fact, same # of infinite emitters)

$\Uparrow$ is still true; $\Downarrow$ doesn't use Cuntz splice



are not flow equivalent
but have the same C^*-alg.

if you remove bottom loop, condition (L) holds
but same remains true

(2) nothing much new to say

(3) $\mathfrak{A} \otimes \mathcal{K} \cong \mathfrak{B} \otimes \mathcal{K}$ $\qquad\qquad$ $\mathfrak{A} \cong \mathfrak{B}$

$\Updownarrow$ $\qquad\qquad$ $\Updownarrow$

$K_*(\mathfrak{A}) \cong K_*(\mathfrak{B})$ $\qquad\qquad$ $(K_*(\mathfrak{A}), [1]) \cong (K_*(\mathfrak{B}), [1])$

Morita equiv. type
classification

currently, can do this well, but not

(4) Have $0 \to C^*(E) \to \mathfrak{A} \to C^*(F) \to 0$

Is $\mathfrak{A}$ a graph $*$-alg?

e.g. $0 \to J \to \mathfrak{A} \to \mathfrak{A}/J \to 0$

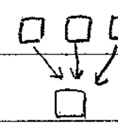
if these are AF, so is $\mathfrak{A}$.

It is not easy to decide if an AF alg is a graph alg

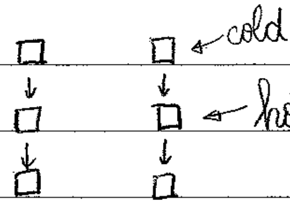
A UHF alg. cannot be a graph alg., but if you stabilize it, it is.

If $\mathfrak{A}$ *is* a graph alg, how is the graph related to E+F?

(6) Conjecture: $FK^+(-)$ is a complete invariant for graph algebras of real rank zero with finitely many ideals.

purely infinite C^*-alg w. ideal lattice 

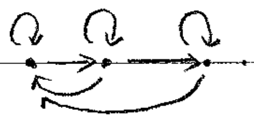
filtered K-theory is not a complete invariant but the algebra cannot be a graph algebra



need to assume K-theory is finitely generated

(7) look for infinite moves that are masa preserving

$A = [\infty]$



are Morita equiv., but it is unclear what moves should be used to get from one to the other. (NOTE: see Mark's webpage for a list of open problems in graph C^*-alg's.)

⑧ $XK(\mathfrak{A}) = \left(K_*(\mathfrak{I})\right)_{\mathfrak{I} \triangleleft \mathfrak{A}}$ + maps

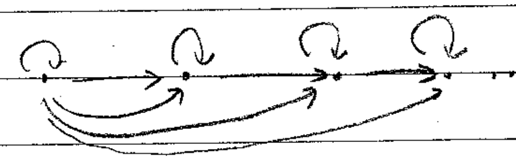
For projective dimension 2, get $\delta \in Ext^2(XK(\mathfrak{A}), \Sigma XK(\mathfrak{A}))$
(XK, δ) is a complete invariant.

⑩ quantum lens spaces

$$C\left(L^q(r; m_1, \dots, m_n)\right) := C(L_q^{2n-1})^{\Lambda}$$ ← fixed pt. alg.

$r \in \mathbb{Z}$, $m_1, \dots, m_n$ - units (mod r)

where $\Lambda : C(L_q^{2n-1}) \to C(L_q^{2n-1})$



$$\varphi(r) = \min\{n \mid (\exists)\ m = (m_1, \dots, m_n) \text{ s.t. } C(L^q(r; \underline{m})) \not\cong C(L^q(r, 1))\}$$
$$= \min\{2n \mid 2n > a > 2 \text{ where } a \mid r\}$$

r	2	3	4	5	6	7	8
	∞	4	6	6	4	8	6.