Fixed Cost Efficiency with Infinitesimal Competitors

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Abstract

Suppose that infinitesimal firms have identical variable costs but there is heterogeneity in their fixed costs. Regardless of the ordering of entry and exit, fixed costs will be minimized for a given industry size.

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1.0 Introduction

Here we consider entry and exit when vanishingly small firms have identical variable cost functions but differ in their exogenous fixed and sunk costs. Suppose that firms differ only in the amount of capital that they must raise to enter (a fraction of which will be sunk). In this case there is the potential for the "wrong" firms to enter the industry. The wrong firms are the firms with the highest entry costs. Further, if the fraction of capital that is not sunk cannot be put to other uses, there is the potential that the high opportunity cost firms will remain in the industry while lower opportunity cost firms will exit.

These fears are unfounded when firm size approaches zero. It is shown that, in this case, all entry orderings will minimize fixed costs. With random shocks to profits and the possibility of exit, the lowest fixed firms enter, when firm size approaches zero. Further, with a continuum of firms it is shown that that the highest opportunity cost firms also exit the industry. Therefore, with infinitesimal firms, only the most efficient competitors remain when low profit realizations drive out potential entrants.

Wilson(2008) shows that under some conditions that the ordering of potential entrants leads to higher fixed and sunk cost firms crowding out lower sunk cost firms. Unlike that paper, this paper considers a continuum of infinitesimally small firms. The present paper finds that high entry cost firms will never crowd out any lower cost rivals. Besides Wilson (2008) the author(s) know(s) of no other study that considers entry orderings effect on the level of fixed costs in an industry when each firm's heterogeneous fixed costs are exogenously determined.

2.0 Epsilon Competitors

We will consider a general model where the size of competitors can be quite small relative to the markups in the industry. Many insights that can be easily gained from calculus are more involved and less transparent if the number of firms must be an integer. Calculus requires that the variable of interest be continuous. In this case, that would mean that entrants are infinitesimally small. Since at least Frank (1965) it has been recognized that the Cournot model predicts that, as number of firms approaches infinity, prices converge to marginal cost. It would be useful to find a way to allow us to use calculus to characterize entry games (as Mankiw and Whinston (1986) does) without having to say this implies that price equals marginal cost.

For this reason, we will have two different measures of the number of entrants. The first measure is of the size of the industry in period 1, N_1. What is unique to this paper is that we have a second measure. This is the number of epsilon, ε, competitors N_1/ε, where $\varepsilon \in [0, N_1]$. While the intensity of competition and the markups over marginal cost are determined by the number of unit-sized competitors, N_1, the actual number of firms entering is the number of epsilon competitors N_1/ε.

The Cournot model can be seen as a special case of the model presented here where $\varepsilon = 1$. A unit-sized competitor is equivalent to $1/\varepsilon$ epsilon competitors. One interpretation of this, when N_1 is an integer, would be to view the $1/\varepsilon$, firms as a collusive group of firms that jointly agree upon output. For example, if there were 10 firms for an industry of size $N_1 = 2$ then ε must be 0.2. In this case, two groups consisting of five firms ($1 \div 0.2 = 5$) would behave as a collusive group. Each group of five firms would jointly determine output. Suppose that the two groups of five then would play a Cournot game. The resulting output would be identical to the case where there were two firms playing a Cournot game. Thus, the Cournot model can be seen as a special case of the model presented here where $\varepsilon = 1$. In the limit, as the size of each epsilon firm approaches zero, the number of epsilon competitors approaches infinity, $\lim_{\varepsilon \to 0} N_1 / \varepsilon \to +\infty$. This may be a reasonable approximation when the number of competitors is very large in an industry, but prices do not converge to marginal cost.

Nevertheless, this analogy of collusive groups is not meant as an explanation of *why* markups may exceed what is suggested by a Cournot-Nash equilibrium. That is, the author does not believe collusive groups are common. The collusive group analogy is used to explain the payoffs from entry, but it is not an explanation of *why* markups are as high or low as they are in equilibrium. Indeed, we assume that all epsilon competitors play a sequential, non-cooperative entry game. Each epsilon-competitor acts to maximize its individual payoff. Instead, we assume that payoffs from this game do not converge to zero as the number of entrants becomes very large.

One plausible explanation of why prices do not converge to marginal cost is that firms produce differentiated products. Other explanations may be the costliness of searching for the lowest cost or most able supplier, respectively. S&P 500 index funds, documented by Hortacsu and Syverson (2004), and real estate agents, documented by Hsieh and Moretti (2003), are examples of competitive environments where prices do not converge to marginal cost after many competitors enter the industry.

[*Insert figure 1 here.***]**

3.0 Model and Analysis

This is a sequential game. The timeline is given in Figure 1. First, the random ordering of entry is determined and becomes common knowledge to all potential entrants in period -1. The ordering of entry is independent of a firm's index number or the random ordering of exit. In period 0 firms sequentially enter based on the order determined in period -1. All entrants must pay a different exogenously determined entry cost. Let us define the entry cost as a function of a firm's index number, N. A firm that enters pays a partially sunk entry cost, $\varepsilon I(N)$, which depends on the firm's index number, N. The investment costs, $\varepsilon I(N)$, are increasing in N. That is, $\frac{\partial I(N)}{\partial N} > 0$. The discount rate between periods is zero. Firms are given the choice to exit in an order revealed at the beginning of time ½, which is unknown at time 0. An epsilon competitor can recover a fraction of its entry cost $\gamma \in [0,1)$. That is, the N-th exiting epsilon competitor receives $\varepsilon\gamma I(N)$ for exiting. Thus $\varepsilon(1-\gamma)I(N)$ is the sunk portion of the entry costs.

In this game the payoffs are random. Let us define s as the state and N_1 as the number of unit sized firms competing in period 1. For any given number of competitors in period 1, N_1, the per-firm payoff before entry costs, $\varepsilon\pi(s, N_1)$, is random when firms must decide to enter in period 0. In period ½ firms learn the state. All firms competing earn identical payoffs, $\varepsilon\pi(s)$, for a given number of firms competing in period 1, N_1. Once the payoffs, $\varepsilon\pi(s, N_1)$, for all possible values of N_1 are revealed, then entrants have the opportunity to exit. This random exit ordering is common knowledge. The stochastic order by which firms can exit is independent of either the index numbers of the firms, the ordering of entry, or the state which determines the payoffs from competition, s. The decisions to exit or stay in the industry are irreversible. All entrants observe the choices of firms to exit or stay in the industry in period ½.

The N-th epsilon competitor that enters in period 0 and stays in period ½ will earn a net payoff after entry costs of $\varepsilon\pi(s, N_1) - \varepsilon I(N)$. For any given state, s', it is assumed that payoff before entry costs is declining in the number of competitors in period 1,

$$\frac{d\pi(s', N_1)}{dN_1} < 0. \qquad (1)$$

Let us suppose that if that the states are smooth and continuous, then the state space is characterized by the probability density function $g(s)$ with the support $[\underline{s}, \bar{s}]$. States, s, are ordered such that profits before entry costs are increasing in the state. Higher states, s, are associated with higher producer surplus for any given N_1. If the distribution of states is continuous, this means that $\pi_s(s, N_1) \equiv \frac{d\pi(s, N_1)}{ds} > 0.$

The number of firms competing in period 1, N_1, cannot exceed the number of entrants in period 0, N^e. Any competitor will weigh the opportunity cost of exiting in period ½ with the expected benefit of staying in the industry. Suppose that at least one epsilon firm exits $N^e > N_1(s)$. Dividing difference between staying in and exiting by ε, the following condition determines the number of firms staying in the industry when the state space is continuous.

$$\pi(s, N_1) - \gamma I(N_1) = 0. \qquad (2)$$

Alternatively, suppose that the state space is discrete. Let T be an integer greater than zero. There are T total states. A given state is denoted by the subscript j. We will order the states, s_j, by the subscript $j = 1, 2, 3, \ldots T$. As in the continuous case, higher states are associated with higher per firm producer surplus. Higher index numbers, higher j's, indicate higher states. That is, $s_1 < s_2 < s_3 < \ldots < s_T$. Per (unit-sized) firm producer surplus is increasing in the index number of the state, j, for any given number of firms, N^*, operating in period 1. That is, $\pi(s_1; N^*) < \pi(s_1; N^*) < \pi(s_3; N^*) \ldots \pi(s_T; N^*)$. The probability of the j-th state is q_j and $\sum_{j=1}^{T} q_j = 1$. When the states are discrete, and the number staying in is less than the number of entrants, $N_1(s_j) < N^e$, then the free exit condition is also a single equality.

$$\pi(s_j; N_1) - \gamma I(N_1) = 0 \tag{3}$$

Let us show that only the firms with the lowest index numbers, $N \in [0, N_1]$, will stay in the industry while the $N > N_1$ indexed firm will prefer to exit in period ½. For a given industry size N_1, all firms with an index number $N \leq N_1$ would weakly prefer to remain in the industry, because $\pi(s; N_1) - \gamma I(N) \geq 0 \;\; \forall N \leq N_1$, when the states are continuously distributed, and $\pi(s_j; N_1) - \gamma I(N) \geq 0 \;\; \forall N \leq N_1$, when the states are discretely distributed. Further, infinitesimally small firms cannot block the entry of other firms because $\pi(N_1) = \pi(N_1 + \varepsilon)$ when $\varepsilon \to 0$. Yet, firms of index numbers $N > N_1$ have higher opportunity costs of staying in because $\frac{dI(N)}{dN} > 0$. Since both equations (2) and (3) bind with equality, higher opportunity costs $\gamma I(N) > \gamma I(N_1)$ implies that all firms with index numbers $N > N_1$ will exit. *Q.E.D.*

Proposition 1

In the limit $\varepsilon \to 0$, there is only one feasible set of firms remaining in the industry after the state of profits has been revealed, regardless of exit ordering. This feasible set is *characterized by all firms with index numbers on the interval* $N \in [0, N_1]$ *staying in and all firms with index numbers* $N > N_1$ *staying out.*

This also implies that expected profits in period 0 are invariant to the random exit orderings revealed in period ½. Instead, expected profits prior to the state being revealed in period ½ only depend the distribution of the profit states. Therefore, we can characterize the number of firms in period 1 as a function of the state $N_1(s)$ with a smooth distribution of states or $N_1(s_j)$ with a discrete state space.

Let us define expected profits for the N-th entrant when the distribution of states is continuous. Let state $s(N) \equiv s^N$ be the lowest state in which the N-th competitor, will stay in the industry in period ½. That is, $\pi(s^N, N) - \gamma I(N) = 0$, or equivalently $N_1(s^N) = N$. $\frac{ds^N(N)}{dN} > 0,$ because higher ranked competitors will only compete in higher states because of their higher opportunity costs. At states $s > s^N$ the N-th firm will choose to remain in the industry and not exit. Expected profits for the N-th epsilon competitor given that it enters when the distribution of states is continuous is $E\{\Pi_C(N)\}$.

$$E\{\Pi_C(N)\} = \varepsilon\left[\int_{s^N}^{\bar{s}} g(s)\pi(s, N_1(s))ds + \gamma I(N)\int_{\underline{s}}^{s^N} g(s)ds - I(N)\right] \tag{4}$$

Expected profits conditional on entry are declining in the firm's index number, N:

$$\frac{\partial E\{\Pi_C(N)\}}{\partial N} = -\varepsilon(1-\gamma G(s^N))\frac{\partial I(N)}{\partial N} < 0 \tag{5}$$

Because $G(s^N) \leq 1$ and $\gamma < 1$ and $\frac{\partial I(N)}{\partial N} > 0,$ the sign of (5) is unambiguously negative. This means expected profits are declining in the firm's index number, N.

Expected profits are also declining in the firm's rank if there is a discrete distribution of states. Let the highest state in which the N-th firm will exit is denoted by $1 \leq X(N) \leq T$. Let $X(N)$ be defined by the following set of inequalities:

$$\begin{aligned} &\pi(s_{X(N)+1}; N_1) - \gamma I(N) \geq 0 \\ &\pi(s_{X(N)}; N_1) - \gamma I(N) < 0 \end{aligned} \tag{6}$$

In this discrete states case, expected profits for the N-th competitor are given by $E\{\Pi_D(N)\}$ below:

$$E\{\Pi_D(N)\} = \varepsilon \left[\sum_{j=X(N)+1}^{T} q_j \pi(s_j; N_1(s_j)) + \gamma I(N) \sum_{j=1}^{X(N)} q_j - I(N) \right] \tag{7}$$

We want to show that firms of higher N, have lower expected profits than lower N firms. There are two cases. In the first case, a firm with higher N will pay a higher entry cost, but it will compete in all the same states that a lower N firm does. It is easy to show and very intuitive that in this first case expected profits are declining in index number. The higher N firm pays a higher entry cost, but does not benefit any more than a lower index number firm from its greater liquidation value. In the second case, the lower index number firm competes in more states than the higher index number firm because the higher N firm liquidates in at least one state in which the lower N firm competes. Here, too, the higher N firm makes lower expected profits, but the proof is somewhat more involved. The intuition is that the higher N firm's gains from liquidating in some low profit states is outweighed by the losses incurred, regardless of the state of having higher entry costs. A formal proof that expected profits decline with firm's index number with a discrete state space is left for the appendix.

The marginal entrant will make zero profits when firm size approaches zero, $\varepsilon \rightarrow 0$. Let us define the marginal entrant as the firm of index number N^e. The marginal entrant, N^e, is determined by equation (8), with continuous states, or equation (9), with discrete states. Free entry when the state space is continuous would mean that

$$E\{\Pi_C(N^e)\} = 0, \tag{8}$$

where the left-hand side is defined in equation (4). Further, free entry when the state space is discrete is given by

$$E\{\Pi_D(N^e)\} = 0, \tag{9}$$

where the left hand side is defined by equation (7).

Since a single competitor cannot affect industry profits by his entry or exit decision when $\varepsilon \rightarrow 0$, sunk cost efficiency can be proven if only the lowest N, lowest sunk cost firms, will enter. Equation (5) shows that expected profits decline in the index number, N, when the distribution of states is continuous; and, the appendix equations (11) and (12) show that expected profits decline in firm index number when the states are finite and discrete. Therefore, only the lowest sunk cost firms, with index numbers N, where $0 \leq N \leq N^e$, will enter when there are random payoffs, exit, and infinitesimal firms. *Q.E.D.*

Proposition 2

With random payoffs from competing in period 1 and infinitesimal competitors, $\varepsilon \rightarrow 0$, only the lowest entry cost competitors will enter, regardless of entry ordering.

This generalization follows from expected profits declining in firms' entry costs $\varepsilon I(N)$.

4.0 Conclusion

When firms have a mass of zero, the continuous case, no firm can block more efficient competitors from entering. Therefore, in this limiting case, only the lowest sunk cost competitors enter the industry, regardless of entry ordering. This result holds when payoffs are random and entrants are allowed to exit after the state of demand is revealed.

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5.0 Appendix: Proof that Expected Profits Decline in Entry Costs when the States are Discrete

Let us define a few terms. $X(N^e) \equiv k,$ where k is a non-negative integer. That is, there are k ordered states. For example, suppose that there are four or more states that the N^e-th firm will exit they are j = 1, 2, 3, .. k. Let $X(N') \equiv k + h$, where h is a non-negative integer, be the number of states in which the N'-th firm will exit. The firm with index number N' exits in h higher states than N^e-th firm. This is due to the fact that $N' > N^e$ and $I(N') > I(N^e),$ and thus the recovery value of the N'-th firms assets are higher, $\gamma I(N') > \gamma I(N^e).$

The expected value of entering the industry are as follows:

$$
\begin{aligned}
E\{\Pi_D(N^e)\} &= \varepsilon\left[\sum_{j=k+1}^{T} q_j \pi(s_j; N_1(s_j)) + \gamma I(N^e)\sum_{j=1}^{k} q_j - I(N^e)\right] \\
E\{\Pi_D(N')\} &= \varepsilon\left[\sum_{j=k+h+1}^{T} q_j \pi(s_j; N_1(s_j)) + \gamma I(N')\sum_{j=1}^{k+h} q_j - I(N')\right]
\end{aligned}
\qquad (10)
$$

In the equation that follows, let us replace the summation of the probabilities of all the states in which the higher ranked firm is liquidated with the parameter $\lambda \equiv \sum_{j=1}^{k+h} q_j$. Because λ is a summation of probabilities, $0 \leq \lambda \leq 1$. With these rearrangements it is clear that the marginal entrant of rank N^e has a higher expected value than any higher N firm that would be liquidated in $h > 0$ more states than the marginal entrant.

$$
\begin{aligned}
&E\{\Pi_D(N^e)\} - E\{\Pi_D(N')\} = \\
&\varepsilon\left\{(1-\gamma\lambda)[I(N') - I(N^e)] + \sum_{j=k+1}^{k+h} q_j[\pi(s_j; N_1(s_j)) - \gamma I(N^e)]\right\} > 0, \qquad (11) \\
&\text{when } h > 0.
\end{aligned}
$$

$\pi(s_j; N_1(s_j)) - \gamma I(N^e) > 0$ for all states where $j = k+1, \ldots k+h$ because these are by definition states where the N^e-th firm finds it profitable to operate. $I(N') - I(N^e) > 0$. Further, $0 \leq \gamma < 1$ and $0 \leq \lambda \leq 1$. Combining all these properties, we can conclude that the marginal entrant is more valuable in expectation than any higher indexed firm that liquidates itself in some higher states.

When $h = 0$ then the difference between the expected value of the N^e-th and N'-th firms is unambiguously positive.

$$
\begin{aligned}
&E\{\Pi_D(N^e)\} - E\{\Pi_D(N')\} = \varepsilon\left[(1-\lambda\gamma)(I(N') - I(N^e))\right] > 0 \qquad (12) \\
&\text{when } h = 0.
\end{aligned}
$$

Since the marginal entrant earns zero profits in expectation, the higher ranked entrant must earn strictly negative profits in expectation. Equations (11) and (12) demonstrate that all firms of index numbers higher than N^e have a strictly lower expected value than the N^e-th firm in period zero. This is what we wanted to show. *Q.E.D.*

Period -1	Period 0	Period ½	Period 1
Ordering of potential entrants is randomly determined and becomes common knowledge.	•Firms sequentially choose to enter. •Entrants pay the sunk costs of entry.	•State of profits are revealed. •Ordering of exit is randomly determined. •Firms sequentially exit. •Exiting firms receive a fraction of their entry cost γ.	Remaining entrants compete.

Figure 1: Sequence of Entry and Exit