Power Calculation for Testing the Mean of a Lognormal Distribution

Let $y_1, ..., y_n$ be a sample of observations from a lognormal distribution with parameters μ and σ^2. Let $x_i = \ln(y_i)$, $i = 1, 2, ..., n$. Note that $\exp(\mu + \sigma^2/2)$ is the mean of the lognormal distribution, and testing the mean is equivalent to testing $\eta = \mu + \sigma^2/2$. Consider

$$H_0 : \eta \geq \eta_0 \quad \text{vs.} \quad H_a : \eta < \eta_0. \tag{1}$$

The sample mean and the variance of the x_i's are respectively given by

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \quad \text{and} \quad s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2.$$

The following algorithm computes the power of the generalized p-value test given in Krishnamoorthy, K. and Mathew, T. (2003, Inferences on the means of lognormal distributions using generalized p-values and generalized confidence intervals, to appear in Journal of Statistical Planning and Inference).

Algorithm 8

For a given μ, σ^2, and the samples size n:
For j = 1, m_1
    Generate $\bar{x}$ from $N(\mu, \sigma^2/n)$ and $s^2 \sim \sigma^2\chi^2_{n-1}/(n-1)$
    For i = 1 to m
        Generate $Z \sim N(0,1)$ and $V^2 \sim \chi^2_{n-1}$
        Set $T_{2i} = \bar{x} - \frac{Z}{V/\sqrt{n-1}}\frac{s}{\sqrt{n}} + \frac{1}{2}\frac{s^2}{V^2/(n-1)}$
        Set $K_i = 1$ if $T_{2i} > \eta_0$, else $K_i = 0$
    (end i loop)
$\frac{1}{m_2}\sum_{i=1}^{m_2} K_i$ is the generalized p-value for testing the hypotheses in (1)
(end j loop)

The proportion of the generalized p-values which are less than the specified nominal level α is the power of the test for testing (1). When $\eta = \eta_0$, the power is equal to α. To compute fairly accurate powers use $m_1 = 2500$ and $m_2 = 5000$.