Hypothesis Testing and Interval Estimation of a Single Lognormal Mean

Let $y_1, ..., y_n$ be a sample of observations from a lognormal distribution with parameters μ and σ^2. Let $x_i = \ln(y_i)$, $i = 1, 2, ..., n$. The sample mean and the variance of the x_i's are respectively given by

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \quad \text{and} \quad s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2.$$

Note that $exp(\mu + \sigma^2/2)$ is the mean of the lognormal distribution, and testing the mean is equivalent to testing $\eta = \mu + \sigma^2/2$. Consider

$$H_0 : \eta \geq \eta_0 \quad \text{vs.} \quad H_a : \eta < \eta_0. \tag{1}$$

Algorithm 1

For a given logged data set, compute the observed sample mean and variance, namely, $\bar{x}$ and s^2, respectively.
For i = 1 to m
Generate a standard normal variate Z
Generate a chi-square random variate V^2 with degrees of freedom $n - 1$
Set $T_{2i} = \bar{x} - \frac{Z}{V/\sqrt{n-1}}\frac{s}{\sqrt{n}} + \frac{1}{2}\frac{s^2}{V^2/(n-1)}$
Set $K_i = 1$ if $T_{2i} > \eta_0$, else $K_i = 0$
(end i loop)
$\frac{1}{m}\sum_{i=1}^{m} K_i$ is the generalized p-value for testing the hypotheses in (1).

If the computed generalized p-value is less than the nominal level α, then the null hypothesis in (1) will be rejected.

The 100(1-α)th percentile of $T_{21}, ..., T_{2m}$, denoted by $T_{2,1-\alpha}$, is the 100(1-α)% generalized upper confidence limit for $\eta = \mu + \sigma^2/2$. Furthermore, $\exp(T_{2,1-\alpha})$ is the $100(1-\alpha)$% generalized upper limit for the lognormal mean. Appropriate quantiles of T_{2i}'s can be used to construct two-sided limits for η or $\exp(\eta)$.

In order to get consistent results regardless of the initial seed used for random number generation, the number of iteration in the above algorithm (i.e., the value of m) should be at least 100,000.