Power Computation for Testing the Difference Between Two Lognormal Means

Consider two lognormal distributions: lognormal(μ_1, σ_1^2) and lognormal(μ_2, σ_2^2). The mean of the ith distribution is given by $\exp(\eta_i)$, where $\eta_i = \mu_i + \sigma_i^2/2$, $i = 1, 2$. Testing

$$H_0 : \exp(\eta_1) \le \exp(\eta_2) \quad \text{vs} \quad H_a : \exp(\eta_1) > \exp(\eta_2) \tag{1}$$

is equivalent to testing

$$H_0 : \eta_1 \le \eta_2 \quad \text{vs} \quad H_a : \eta_1 > \eta_2. \tag{2}$$

For given sample sizes n_1 and n_2, and the parameters (μ_1, σ_1^2) and (μ_2, σ_2^2), the Monte Carlo method given in the following algorithm can be used to compute the power of the generalized test (due to Krishnamoorthy, K. and Mathew, T. (2003). Inferences on the means of lognormal distributions using generalized p-values and generalized confidence intervals, to appear in Journal of Statistical Planning and Inference) for testing hypotheses in (2).

Algorithm

For i = 1, m_1
  Generate $\bar{x}_1 \sim N(\mu_1, \sigma_1^2/n_1)$ and $\bar{x}_2 \sim N(\mu_2, \sigma_2^2/n_2)$
  Generate $s_1^2 \sim \sigma_1^2 \chi^2_{n_1-1}/(n_1-1)$ and $s_2^2 \sim \sigma_2^2 \chi^2_{n_2-1}/(n_2-1)$
For j = 1 to m_2
  Generate Z_1 and Z_2 from $N(0,1)$
  Generate $V_1^2 \sim \chi^2_{n_1-1}$ and $V_2^2 \sim \chi^2_{n_2-1}$
  Compute $T_j = (\bar{x}_1 - \frac{Z_1}{V_1/\sqrt{n_1-1}} \frac{s_1}{\sqrt{n_1}} + \frac{1}{2} \frac{s_1^2}{V_1^2/(n_1-1)}) - (\bar{x}_2 - \frac{Z_2}{V_2/\sqrt{n_2-1}} \frac{s_2}{\sqrt{n_2}} + \frac{1}{2} \frac{s_2^2}{V_2^2/(n_2-1)})$
  Set $K_j = 1$ if $T_j < 0$, else $K_j = 0$
(end j loop)
$\frac{1}{m_2} \sum_{j=1}^{m_2} K_j$ is the generalized p-value for testing the hypotheses in (2).
If the generalized p-value $< \alpha$, then set $Q_i = 1$; else set $Q_i = 0$
(end i loop)
$\sum_{i=1}^{m_1} Q_i/m_1$ is a Monte Carlo estimate of the power.

Satisfactory accuracies can be attained if $m_1 = 2500$ and $m_2 = 5000$ are used.