

# Power Computation for Testing the Difference Between Two Lognormal Means

Consider two lognormal distributions:  $\text{lognormal}(\mu_1, \sigma_1^2)$  and  $\text{lognormal}(\mu_2, \sigma_2^2)$ . The mean of the  $i$ th distribution is given by  $\exp(\eta_i)$ , where  $\eta_i = \mu_i + \sigma_i^2/2$ ,  $i = 1, 2$ . Testing

$$H_0 : \exp(\eta_1) \leq \exp(\eta_2) \quad \text{vs} \quad H_a : \exp(\eta_1) > \exp(\eta_2) \quad (1)$$

is equivalent to testing

$$H_0 : \eta_1 \leq \eta_2 \quad \text{vs} \quad H_a : \eta_1 > \eta_2. \quad (2)$$

For given sample sizes  $n_1$  and  $n_2$ , and the parameters  $(\mu_1, \sigma_1^2)$  and  $(\mu_2, \sigma_2^2)$ , the Monte Carlo method given in the following algorithm can be used to compute the power of the generalized test (due to Krishnamoorthy, K. and Mathew, T. (2003). Inferences on the means of lognormal distributions using generalized p-values and generalized confidence intervals, to appear in Journal of Statistical Planning and Inference) for testing hypotheses in (2).

## Algorithm

For  $i = 1, m_1$

    Generate  $\bar{x}_1 \sim N(\mu_1, \sigma_1^2/n_1)$  and  $\bar{x}_2 \sim N(\mu_2, \sigma_2^2/n_2)$

    Generate  $s_1^2 \sim \sigma_1^2 \chi_{n_1-1}^2 / (n_1 - 1)$  and  $s_2^2 \sim \sigma_2^2 \chi_{n_2-1}^2 / (n_2 - 1)$

For  $j = 1$  to  $m_2$

    Generate  $Z_1$  and  $Z_2$  from  $N(0, 1)$

    Generate  $V_1^2 \sim \chi_{n_1-1}^2$  and  $V_2^2 \sim \chi_{n_2-1}^2$

    Compute  $T_j = (\bar{x}_1 - \frac{Z_1}{V_1/\sqrt{n_1-1}} \frac{s_1}{\sqrt{n_1}} + \frac{1}{2} \frac{s_1^2}{V_1^2/(n_1-1)}) - (\bar{x}_2 - \frac{Z_2}{V_2/\sqrt{n_2-1}} \frac{s_2}{\sqrt{n_2}} + \frac{1}{2} \frac{s_2^2}{V_2^2/(n_2-1)})$

    Set  $K_j = 1$  if  $T_j < 0$ , else  $K_j = 0$

(end j loop)

$\frac{1}{m_2} \sum_{j=1}^{m_2} K_j$  is the generalized p-value for testing the hypotheses in (2).

If the generalized p-value  $< \alpha$ , then set  $Q_i = 1$ ; else set  $Q_i = 0$

(end i loop)

$\sum_{i=1}^{m_1} Q_i / m_1$  is a Monte Carlo estimate of the power.

Satisfactory accuracies can be attained if  $m_1 = 2500$  and  $m_2 = 5000$  are used.