5.8 Problems (last update 30 May 2018)

1.The lineup or batting order for a baseball team is a list of the nine players on the team indicating the order in which they will bat during the game.

a) How many lineups are possible?

b) If lineup is restricted so that the catcher is listed first and the pitcher is listed last, how many lineups are possible?

2. In how many ways can one plumber, one electrician, and one carpenter be selected when there are 5 choices of plumber, 3 choices of electrician, and 7 choices of carpenter?

3. (Akritas) The clock rate of a CPU (central processing unit) chip refers to the frequency, measured in megahertz (MHz), at which it functions reliably. CPU manufacturers typically categorize (bin) CPUs according to their clock rate and charge more for CPUs that operate at higher clock rates. A chip manufacturing facility will test and bin each of the next 10 CPUs in four clock rate categories denoted by G1, G2, G3, and G4.

a) How many possible outcomes of this binning process are there?

b) How many of the outcomes have three CPUs classified as G1, two classified as G2, two classified as G3, and three classified as G4?

c) If the outcomes of the binning process are equally likely, what is the probability of the event described in part (b)?

4. A president, treasurer, and secretary, all different, are to be chosen from a club consisting of 10 people. How many different choices of officers are possible if

a) There are no restrictions?

b) A and B will not serve together?

c) C and D will serve together or not at all?

d) E must be an officer?

e) F will serve only if he is president?

5. A child has 12 blocks, of which 6 are red, 3 are green, 2 are blue, and 1 is black. If the child puts the blocks in a line, how many arrangements are possible?

6. If 4 Americans, 3 French people, and 3 British people are to be seated in a row.

a) How many seating arrangements are possible for these 10 individuals when there is no restriction on where each person may be seated?

b) How many seating arrangements are possible when people of the same nationality must be seated next to each other?

c) How many seating arrangements are possible when the 4 Americans must sit next to each other but there is no restriction on the others?

7. For years, telephone area codes in the United States and Canada consisted of a sequence of three digits. The first digit was an integer from 2 through 9; the second digit was either 0 or 1; and the third digit was any integer from 1 through 9.
a) How many area codes were possible?
b) How many area codes ending in 6 were possible?

8. In how many ways can 4 novels, 3 biology books, and 2 mathematics books be arranged on a bookshelf if
a) the books can be arranged in any order;
b) the biology books must be together and the novels must be together;
c) the novels must be together but the other books can be arranged in any order?

9. Two pollsters will canvas a neighborhood with 20 houses. Each pollster will visit 10 houses. No house will be visited more than once. How many different assignments of pollsters to houses are possible?

10. A box contains 24 light bulbs, of which two are defective. If a person selects 10 light bulbs at random, without replacement, what is the probability that both defective light bulbs will be selected?

11. Suppose that two defective refrigerators have been included in a shipment of six refrigerators. Now suppose that the buyer begins to test the refrigerators one at a time.
a) What is the probability that the last defective refrigerator is found on the fourth test?
b) What is the probability that no more than four refrigerators need to be tested before both defective refrigerators are found?

12. Consider two collections of 4 cards, one that contains 4 red cards numbered from 1 to 4 and one that contains 4 blue cards numbered from 1 to 4. Suppose that one card is selected at random from the 4 red cards and one card is selected at random from the 4 blue cards.
Let A denote the event that the number on the red card is larger than the number on the blue card.
Let B denote the event that the number on the red card is greater than two.
Let C denote the event that the number on the blue card is odd.

a) List the 16 possible outcomes of this experiment as ordered pairs with the first element representing the number on the red card and the second element representing the number on the blue card, *e.g.* the ordered pair $(1, 4)$ indicates that the red card numbered 1 was selected and the blue card numbered 4 was selected.

Using this collection of 16 ordered pairs as a representation of the sample space Ω, describe each of the following events both in words and as subsets of Ω:

b) $A \cap B$
c) $A \cup C$
d) $B \cap (A \cup C)$
e) $B^c \cap C^c$
f) $A \cap B \cap C$.

13. This problem is concerned with a simple visual messaging system based on an ordered arrangement of ten colored flags. A message is formed by placing ten colored flags on a single flagpole. The message is read by noting the colors of the flags starting at the top of the pole and moving down. Suppose that the ten flags consist of two red flags, three blue flags, and five green flags.

a) How many different messages can be formed from these ten flags?
b) How many different messages can be formed from these ten flags if the first and last flags must be red?
c) How many different messages can be formed from these ten flags if the fifth and sixth flags must be red?
d) How many different messages can be formed from these ten flags if the every other flag, starting with the first must be green?

14. Consider three groups, group A, group B, and group C, each consisting of ten people. A subgroup of three people is to be selected from the combined group of thirty people.

a) How many choices of three people are possible?
b) How many choices of three people are there in which all three people come from group A?
c) How many choices of three people are there in which all three people come from the same group of ten?
d) How many choices of three people are there in which one of the three people comes from group A, one comes from group B, and one comes from group C?

15. Suppose that five cards are selected at random without replacement from a standard deck of 52 playing cards.

a) Find the probability that the five cards selected include exactly three face cards (Jack, Queen, King).
b) Find the probability that the five cards selected include exactly three hearts and exactly two spades.
c) Find the probability that the five cards selected include exactly two clubs and exactly two diamonds.
d) Find the probability that the five cards selected consist of three sevens and two non–sevens which are not of the same kind.

16. A box contains 100 balls of which 20 are red, 30 are white, and 50 are blue. If 6 balls are selected at random with replacement from this box, find the following.

a) The probability that exactly 3 of the 6 balls are red
b) The probability that 2 of the 6 balls are red and 4 are white.
c) The probability that 1 of the 6 balls is red, 2 are white, and 3 are blue.

17. **This is the without replacement version of problem 16.** A box contains 100 balls of which 20 are red, 30 are white, and 50 are blue. If 6 balls are selected at random without replacement from this box, find the following.

a) The probability that exactly 3 of the 6 balls are red
b) The probability that 2 of the 6 balls are red and 4 are white.
c) The probability that 1 of the 6 balls is red, 2 are white, and 3 are blue.

18. Consider an elevator with five passengers. Suppose that this elevator stops at ten floors. Also assume that each passenger chooses the floor at which he or she gets off at random and independently of the other passengers. Find the probability that no two passengers get off at the same floor. **Hint:** You can think of the assignment of floors to passengers as the selection, at random and with replacement, of 5 numbers from the set $\{1, 2, \ldots, 10\}$.