- The title of my talk today is "Symmetric spectra: the objects that give rise to generalized cohomology theories."
- I'd like to start by saying that every generalized cohomology theory comes from a symmetric spectrum in a canonical way, and every symmetric spectrum gives a generalized cohomology theory, again in a canonical way.
- For example, if $H^*(-)$ is a generalized cohomology theory, then, roughly speaking, there is a symmetric spectrum E such that for every pointed CW-complex X,

$$H^*(X) = [\Sigma^\infty X, E]_*,$$

where, roughly speaking, the right-hand side is the set of homotopy classes of morphisms between the symmetric spectra $\Sigma^\infty X$ and E, and $\Sigma^\infty X$ can be thought of as the collection of topological spaces

$$\{X, S^1 \wedge X, S^1 \wedge S^1 \wedge X, ...\}.$$

($S^1 \wedge X$ is the "reduced suspension" of X.)
- I mention this as motivation for why symmetric spectra are important, but for now at least, I don't want to say anything more about this.
- For me, the main motivation for this talk is that spectra are the basic objects that I work with in my research. I just said "spectra," instead of symmetric spectra and my title says "the objects," so I need to do some explaining. There are various categories of spectra and, in terms of the relation of homotopy, they are all equivalent to each other. I regularly work with three different categories of spectra and each has its advantages and disadvantages.
- In maybe one seminar talk and one, maybe two Colloquia, I've quickly given a definition of spectrum, but it was for a category of spectra that (a) was very easy to define, (b) had some serious disadvantages, and (c) is not suitable for much of the serious work that is being done these days. What one really wants to work with is a closed symmetric monoidal category of spectra. After the desire for such a category arose, it took several decades before one was constructed. The category of symmetric spectra is (a) one such category, (b) it's the easiest of these to define, and (c) because it uses simplicial sets instead of topological spaces, it's the best one to use in my research.
- The next thing I want to do is explain what a closed symmetric monoidal category is, but first I'd like to explain why such a

category is important for spectra. It allows for certain algebraic constructions with spectra that are otherwise impossible: for example, in the category of symmetric spectra, there is an analogue of the notion of a commutative ring and of a commutative algebra over such a commutative ring object. The stable homotopy groups of a commutative ring object in the category of symmetric spectra are a graded commutative ring.