The Distribution of Stars Most Likely to Harbor Intelligent Life

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Abstract

Simple heuristic models and recent numerical simulations show that the probability of habitable planet formation increases with stellar mass. We combine those results with the distribution of main sequence stellar masses to obtain the distribution of stars most likely to possess habitable planets as a function of stellar lifetime. We then impose the self selection condition that an intelligent observer can only find themselves around a star with a lifetime greater than the time required for that observer to have evolved, T_i. This allows us to obtain the stellar timescale number distribution for a given value of T_i. Our results show that for habitable planets with a civilization that evolved at time $T_i = 4.5$ Gyr the median stellar lifetime is 13 Gyr, corresponding approximately to a stellar type of G5; with 2/3 of the stars having lifetimes between 7 and 30 Gyr, corresponding approximately to spectral types G0 - K5. For other values of T_i the median stellar lifetime changes by less than 50%.

1 Introduction

In the ongoing search for extrasolar habitable planets and the SETI search for intelligent life it is desirable to know the optimum target star types. The principle of mediocrity (aka Copernican Principle) holds that, in the absence of any contrary evidence, we should assume that our solar system is typical among those planetary systems with intelligent observers. In particular this suggests that the sun is a typical star among those stars supporting habitable planets and intelligent life. Thus most SETI and extrasolar planet searches have favored solar type stars. On the other hand, lower mass stars, especially M stars, are much more numerous (therefore closer) and have been the focus of recent attention in the search for habitable planets (Tarter, *et al.* 2007; Scalo *et al.* 2007).

In this paper we attempt to quantify the distribution of stars most likely to support habitable planets and therefore the possibility of intelligent life. In section 2 we model the probability of habitability as a function of stellar mass. We combine this with the stellar distribution of main sequence lifetimes in section 3 to obtain the distribution of stars

most likely to possess habitable planets. In section 4 we impose the requirement that the habitable star lifetime must exceed the time required for intelligence to evolve T_i (= age of the star when intelligence evolved). This allows us to obtain the stellar distribution as a function of T_i. We discuss parameter variations in section 5 and summarize our conclusions in section 6.

2　The stellar habitability weighting function

We wish to model the probability, $P(M)$, that a star of given mass M has one or more terrestrial planets within its habitable zone (HZ). The HZ around a star is the radial band Δr_{HZ} within which an Earth-like planet could support surface liquid water. The inner and outer HZ radii are determined by the critical stellar fluxes associated with the loss of water and permanent glaciation (Kasting *et al.* 1993). In the following analysis it is not crucial to distinguish between zero age main sequence, mid-main sequence, or specific continuously-habitable zones. We follow Raymond, Scalo and Meadows (2007, hereafter RSM) and assume a minimum planetary habitability mass of 0.3 M$_{\oplus}$.

In the planet formation process, the number of hypothetical Hill spheres, which roughly determines the potential number of accreting embryos, contained within Δr_{HZ} increases with decreasing stellar mass. This theoretical expectation is supported by simulations. However, the final planet mass depends primarily on the total mass available in the HZ annulus, M_{HZ}, and not on the total number of embryos. Kokubo *et al.* (2006) found that the mean planet mass M_{pl} scaled linearly with disk mass $M_{\mathrm{pl}} \propto M_{\mathrm{disk}}^{0.97-1.1}$. RSM found that the mean planet mass $M_{\mathrm{pl}} \propto M_{HZ}^{1.1}$. Following RSM, we assume the surface density of protoplanetary disks, $\sigma(r)$, varies with heliocentric distance r, as

$$\sigma(r) = \sigma_1 F Z \left(\frac{r}{1\mathrm{AU}}\right)^{-\alpha} \left(\frac{M}{\mathrm{M}_{\odot}}\right)^h \tag{1}$$

where M is the mass of the star, σ_1 is the surface density of solids at 1 AU in the minimum-mass solar nebular model, F is a scaling factor for the disk mass ($F = 1$ for the minimum mass solar nebula), $Z = 1$ is the solar metallicity and α is the disk density radial power index. Using the fiducial parameters of RSM, $F = Z = \alpha = h = 1$ leads to the predicted scaling for the planetary mass as a function of M,

$$M_{\mathrm{pl}} \propto \sigma(r_{HZ})r_{HZ}\Delta r_{HZ} \propto M\Delta r_{\mathrm{HZ}} \propto ML^{\frac{1}{2}} \propto M^{1+\frac{p}{2}}, \tag{2}$$

where L is the stellar luminosity and we have set r_{HZ} (the mean HZ radius) $= r$ in Eq. (1). These proportionalities follow from the observation that both r_{HZ} and Δr_{HZ} are $\propto L^{1/2}$ for a fixed critical flux. L is related to M by the mass-luminosity relation $L \propto M^p$.

This simple model was supported by the late stage accretion simulations of RSM. They concluded that the fraction of systems with sufficient disk mass to form $\geq 0.3\mathrm{M}_{\oplus}$ habitable planets decreases for low mass stars for every realistic combination of parameters. A similar conclusion, based in part on volatile deficiency of HZ planets around low mass stars, was reached by Lissauer (2007). Using RSM's fiducial parameters, the

mean (determined by averaging over several simulations) planetary mass dependency on stellar mass M is approximately $M_{\rm pl} \propto M^3$. We assume as our working heuristic model that the probability $P(M)$ is also proportional to the mass in the HZ annulus $P(M) \propto M^{1+\frac{p}{2}}$.

Figure 6 of RSM gives the fraction of simulations, $P(M)$, for which stars of a given mass produce one or more HZ planets of critical mass $\geq 0.3 M_\oplus$ as a function of M for 0.2 M$_\odot < M < 1.0$ M$_\odot$ and for several values of α. The other parameters are fixed at their fiducial values $F = Z = h = 1$. We approximate these curves with power law fits, $P(M) \propto M^a$. The fiducial case of $\alpha = 1$ can be fit with $a = 3$, in agreement with our heuristic model for $p=4$. In a simple model in which the probability of a star possessing at least one habitable planet is assumed to be proportional to the absolute width of the habitable zone, $P(M) \propto \Delta r_{HZ} \propto L^{\frac{1}{2}} \propto M^{\frac{p}{2}}$, $a = 2$ for $p = 4$. In a model in which $P(M)$ is proportional to the number of Hill radii times the annular mass contained in the HZ, $a = 3$. In the following analysis we treat a as a parameter independent or p,

$$P(M) = \begin{cases} (\frac{M}{\rm M_\odot})^a & M < 1{\rm M_\odot} \\ \\ 1 & M > 1{\rm M_\odot} \end{cases} . \tag{3}$$

The value of $P(M) = 1$ for $M > 1{\rm M_\odot}$ is based on a rough extrapolation of RSM's simulations for their $\alpha =1$ case (our $a = 3$) beyond their simulation cut off mass of $1{\rm M_\odot}$. By inspection of this curve and the trend of their $M_{\rm critical} = 0.1 M_\oplus$ curve it is evident that $P(M)$ asymptotically approaches 1.0 somewhat beyond $M = 1{\rm M_\odot}$.

3 The number distribution of "habitable" stars

3.1 The number distribution in mass

The number distribution of stellar masses $\phi(M) \equiv (dN/dM)$ when multiplied by $P(M)$ gives the number distribution of stars weighted according to their habitability probability, or the number distribution of "habitable stars" for brevity, $\phi_H(M) \equiv \phi(M)P(M) = (dN_H/dM)$. The stellar mass distribution can be approximated by a power law over a given range of masses, $\phi(M) \propto M^{-q}$. For stellar masses less than $1{\rm M_\odot}$ a power law index of $q = 1.0\pm 0.5$ is consistent with most studies (Massey and Meyer 2001). Over this range of masses there is little difference between the present day mass function (PDMF) and the initial mass function (IMF). For stars of mass $> 1{\rm M_\odot}$ an adequate approximation is the Salpeter IMF with index $q' = 2.35$ and the PDMF with index $q' = 5.4$ (Charbier 2003). In the simulations of RSM (their Fig. 6) $P(M)$ also has a break near $M \approx 1{\rm M_\odot}$, beyond which $P(M) \approx 1$. In the following analysis stellar masses and lifetimes are in solar units, M$_\odot \equiv 1$ and T$_\odot \equiv 1$. The number distribution in mass of *habitable stars* is then

$$\phi_H(M) \propto \begin{cases} M^{a-q} & M < 1 \\ M^{-q'} & M > 1 \end{cases} . \tag{4}$$

For $a = 3$, $q = 1$, and $q' = 5.4$, $\phi_H(M)$ is strongly peaked at 1 M$_\odot$.

3.2 The distribution in main sequence lifetime

Assuming a mass-luminosity relation of the form $L \propto M^p$ and a main sequence lifetime $T \propto (M/L) \propto M^{1-p}$, one obtains

$$M \propto T^{\frac{1}{1-p}} \tag{5}$$

and

$$\frac{dM}{dT} \propto T^{\frac{p}{1-p}}. \tag{6}$$

The number distribution of habitable stars in main sequence lifetime $\phi_H(T) \equiv (dN_H/dT) = \phi_H(M)(dM/dT)$ is

$$\phi_H(T) \propto \begin{cases} T^{\frac{a+p-q}{1-p}} \equiv T^s & T > 1 \\ T^{\frac{p-q'}{1-p}} \equiv T^{s'} & T < 1 \end{cases} . \tag{7}$$

4 The number distribution of "intelligent" stars

We assume that the number distribution of "intelligent" stars will be proportional to the distribution of habitable stars given by Eq. (7) but truncated at $T = T_i$, since the region $T \leq T_i$ is logically excluded. We neglect any other (unknown) dependencies of T_i on T. In our model the joint number distribution is

$$\phi(T_i, T) \propto \begin{cases} 0 & T_i > T \\ \phi_H(T)\phi_i(T_i) & T_i < T \end{cases} . \tag{8}$$

where the unknown number distribution $\phi_i(T_i)$ and $\phi_H(T)$ are assumed independent and therefore separable in the joint distribution.

To help clarify our analysis we first consider a hypothetical thought experiment. We imagine that there is a Galactic central repository that contains data on all intelligent civilizations that have ever existed in the history of the Galaxy. Included in this data base are the times of emergence of these intelligent civilizations, T_i, and the main sequence lifetime of their local star, T. We now imagine querying the repository for a list of pairs (T_i, T) of stars currently in their main sequence phase which have, or previously had, an intelligent civilization. It is this hypothetical data base that our statistical analysis attempts to model.

Assume we have a collection of habitable stars with intelligent life evolving at the *same* time T_i to $T_i + \Delta T_i$. We can use Eq. (8) to find the fraction, f, of these intelligent stars with lifetimes in the interval between $T_{\min}$ and T_f and the fraction (1-f) with lifetimes in the interval between T_f and $T_{\max}$,

$$\frac{f}{1-f} = \frac{\phi_i(T_i)\Delta T_i \int_{T_i}^{T_f} \phi_H(T)dT}{\phi_i(T_i)\Delta T_i \int_{T_f}^{T_{\max}} \phi_H(T)dT}, \tag{9}$$

where the unknown number distribution $\phi_i(T_i)$ cancels and the lower limit $T_{\min}$ has been set equal to T_i, the time required for the evolution of the intelligent observers. Thus $T_f \equiv T_f(T_i)$ is the lifetime of that intelligent star which divides the distribution into

fractions f and $(1 - f)$ for the specific T_i. For example, if $f = 1/2$, the corresponding $T_{f=1/2}$ is the median lifetime of the distribution. Assuming first that $T_f \leq 1$,

$$\frac{1-f}{f} \int_{T_i}^{T_f} T^{s'} dT = \int_{T_f}^{1} T^{s'} dT + \int_{1}^{T_{\max}} T^s dT \tag{10}$$

where s and s' are defined in Eq. (7). For Earth $T_i^{\oplus} = 0.45$, in solar units. The intelligent star distribution is truncated at $T_{\min} = T_i$ (= age of star when intelligent life evolved) because an observer cannot be found around a star with a lifetime less than their own T_i. Since $\phi_i(T_i)$ is assumed to be independent of $\phi_H(T)$, the lower limit of T_i can be treated as a constant for the purpose of the integration. Solving Eq. (10) yields

$$T_f = \left[\left(\frac{1-f}{f} \right) T_i^{s'+1} + 1 - \left(\frac{s'+1}{s+1} \right) + \left(\frac{s'+1}{s+1} \right) T_{\max}^{s+1} \right]^{\frac{1}{s'+1}} f^{\frac{1}{s'+1}}. \tag{11}$$

If $T_f \geq 1$, the fraction, f, of habitable stars with lifetimes $\leq T_f$ is then obtained from

$$\int_{T_i}^{1} T^{s'} dT + \int_{1}^{T_f} T^s dT = \frac{f}{1-f} \int_{T_f}^{T_{\max}} T^s dT \tag{12}$$

and

$$T_f = \left[\left(\frac{s+1}{s'+1} \right) T_i^{s'+1} + 1 - \left(\frac{s+1}{s'+1} \right) + \left(\frac{f}{1-f} \right) T_{\max}^{s+1} \right]^{\frac{1}{s+1}} (1-f)^{\frac{1}{s+1}}. \tag{13}$$

5   Discussion

For a given set of parameters (a, q, q', and p) Eqs. (11) and (13) can be used to find the median stellar lifetime, $T_{f=1/2}$, of habitable stars having intelligent life that evolved at time T_i, and the approximate $\pm 1\sigma$ lifetimes, $T_{f=1/6}$ and $T_{f=5/6}$. Since $T_{\max}(\approx 10^3) \gg T_i(\leq 1)$ terms involving $T_{\max}$ are negligible for all realistic parameters. (There may be other, *e.g.* geological, stronger limits on $T_{\max}$ and this will be considered elsewhere.) We adopt fiducial parameters $a = 3, q = 1, q' = 5.4$, and $p = 3.5$, which yield $s = -2.2$ and $s' = 0.76$. The value of $a = 3$ is based on a power law fit to RSM's $\alpha = 1$, $M_{\text{critical}} = 0.3 M_{\oplus}$ case, *i.e.* their fiducial set of parameters. Our choice of $q = 1$ is an adequate approximation (Massey and Meyer 2001) of more complex fits to the data (Chabrier 2003) and $q' = 5.4$ is the present day mass function exponent. Finally, p = 3.5 is a compromise of a range of power laws with exponents ranging from 2.7 for the least massive stars to 4.7 for solar mass stars to 3.0 for more massive stars (Massey & Meyer 2001).

Figure 1 is based on our fiducial parameters and shows T_f as a function of the time of appearance of intelligent life T_i. For a given T_i the thick solid curve gives the corresponding median stellar lifetime, $T_{f=1/2}$, from Eqs. (11) and (13). The two thin solid curves are for $T_{f=1/6}$ and $T_{f=5/6}$ in Eqs. (11) and (13) and are approximately the $\pm 1\sigma$ curves. For a given T_i these curves bracket the most probable associated stellar lifetimes. From Eq. (9), the $f = 0$ dashed curve corresponds to $T_{f=0} = T_i$ and, since

intelligent life must evolve before the end of the life of the star, the phase space below this curve is excluded. The solar system data is indicated by the $\oplus$ symbol. The theoretical distribution $\phi_i(T_i)$ is unknown, but realizable T_i's must be less than the age of the Galaxy, $\text{Log}(T_{\text{gal}}) = 0$. We note that the median and $\pm 1\sigma$ brackets for *habitable* stars only, $\phi_H(T)$, can be found from the figure by setting $T_i = 0$, or $\text{Log}(T_i) = -\infty$. The figure shows that there is not a significant difference in the number distribution curves for *habitable* stars (only) and for "intelligent" stars unless $\text{Log}(T_i) > -0.5$.

If we searched the previously discussed Galactic repository data base for civilizations having the *specific* timescale $T_i = T_i^{\oplus}$ we would record a distribution of stellar lifetimes given by $\phi_H(T > T_i^{\oplus})$, *i.e.* Eq. (7) truncated at $T_i^{\oplus}$. From Eqs. (11) and (13) or from Fig. 1 we find that half of the civilizations in this data base will have reported that their local star has a lifetime ≥ 1.26 and half will have reported that their local star has a lifetime ≤ 1.26. Two thirds will have reported that their local star has a lifetime lying somewhere between 0.72 and 3.1.

Varying each parameter over a realistic range while holding the other fiducial parameters and $T_i = 0.45$ fixed showed that our results are robust. The test ranges were as follows: $a = 1.5 \leftrightarrow 4.0, q = 0.5 \leftrightarrow 1.5, q' = 2.35 \leftrightarrow 5.4$, and $p = 2.5 \leftrightarrow 4.7$. The minimum $T_{f=1/2}$ is 1.1 and the maximum is 2.1. This maximum corresponds to $a = 1.5$ ($\alpha \approx 3/2$). This is the original MMSN value of α (Hayashi 1981) but more recent theoretical and observational studies suggest that $\alpha = 1/2 \leftrightarrow 1$ (Davis 2005; Garaud & Lin 2007; Andrews and Williams 2007; RSM). Values of α between 1/2 and 1 correspond to our $a \approx 3$ and 4, yielding $T_{\frac{1}{2}} = 1.26$ and 1.1, respectively.

The approximate equality of T_i and T for the solar system along with the assumption that $\phi(T)$ and $\phi_i(T_i)$ are independent led Carter (1983) to conclude that intelligent life is rare in the universe. However, this "coincidence" cannot apply to most stars: a typical T for an M star (*e.g.*, 1,000 Gyr) is much greater than T_i since T_i is limited by the age of the Galaxy (10 Gyr). The solar system coincidence could be rephrased - why don't we find ourselves around a more typical star? Figure 1 shows that most intelligent life should arise around stars of mass $\approx 1 \text{ M}_\odot$. With the additional (plausible) assumption that the probability of the evolution of intelligent life increases with time, the solar system data can be understood separately from the Carter rarity argument.

6 Conclusions

Our main results are summarized in Figure 1 which shows the most probable stellar lifetimes as a function of the time of appearance of intelligent life, T_i. The median stellar lifetime associated with an intelligent civilization that evolved in time $T_i = 4.5$ Gyr is 12.6 Gyr, corresponding to a spectral type of approximately G4, with $\approx \pm 1\sigma$ range of K5 - G0. For different values of T_i the corresponding stellar lifetimes vary somewhat but are within 50% of the solar system value.

FIGURE CAPTION

Fig. 1 T_f as a function of T_i in solar units. Here T_i is the time of appearance of a set of intelligent observers orbiting habitable stars with lifetimes $T > T_i$. The fraction of these habitable stars with lifetimes between T_i and T_f is f. That is, T_f is the lifetime of the habitable star that divides the distribution into fractions f and $(1-f)$. The thick solid curve corresponds to $f = 1/2$ in Eqs. (11) and (13) and gives the median intelligent star lifetime as a function of the *specified* T_i; *i.e.*, in the set consisting of all civilizations that have the same given biological timescale T_i, half will find themselves around stars of lifetime $\geq T_{f=1/2}$ and half will find themselves around stars of lifetime $\leq T_{f=1/2}$. The two thin solid curves are for $f = 1/6$ and $5/6$ and approximately bracket the $\pm 1\sigma$ region. Thus for a given T_i, approximately 2/3 of the local stars will have a lifetime, T, lying between $T_{f=1/6}$ and $T_{f=5/6}$, the region noted as Most Probable in the figure. From Eq. (9), the $f = 0$ dashed line corresponds to $T_f = T_i$ and, since intelligent life must evolve before the end of the life of the star, the phase space below this curve is excluded and noted Impossible in the figure. Correspondingly, $f = 1$ implies $T_f = T_{\max} = 10^3$. The location of the solar system is indicated by the $\oplus$ symbol and it can be seen that the sun lies near the median for $T_i = 0.45$. Though the number distribution $\phi_i(T_i)$ is unknown, realizable T_i's must be less than the age of the Galaxy, $\mathrm{Log}(T_{\mathrm{gal}}) = 0$.

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