

The Pioneer 10 and 11 anomalous acceleration and  
Oort cloud comets

Daniel P. Whitmire and John J. Matese

Department of Physics, University of Louisiana at Lafayette

Submitted 21 January 2003

Revised form 18 April 2003

Submitted to Icarus

Manuscript Pages: 12

Figures: 2

**Proposed Running Head:** Pioneer Anomalous Acceleration

**Editorial Correspondence to:**

Dr. Daniel P. Whitmire

Department of Physics

University of Louisiana at Lafayette

70504-4210 USA

Phone:(337)482 6185

Fax: (337) 482 6699

Email: whitmire@louisiana.edu

**Abstract:** Anderson *et al.* (2002) recently reported new evidence that Pioneer 10 and 11 are experiencing an unmodeled anomalous acceleration directed toward the Sun. Numerous mechanisms, both internal and external to the spacecraft, have been proposed to explain this unmodeled acceleration. If we assume that the cause of the acceleration is (1) external to the spacecraft, (2) isotropic and (3) acts on bodies of cometary mass, then it would imply that new comets are more tightly bound to the Sun than previously believed. Here we show that the implied higher binding energy is incompatible with the established evidence that the galactic tide is dominant in making Oort cloud comets observable. We conclude that one or more of these assumptions must be invalid.

*Key Words:* comets, dynamics; celestial mechanics

# 1 Introduction

Anderson *et al.* (2002) have given a comprehensive update and review of the Pioneer 10 and Pioneer 11 anomalous acceleration problem. The new results extend the radial distance over which the unmodeled acceleration is observed from 20 to 70 AU. They find that the anomalous acceleration is  $a_p = (8.74 \pm 1.33) \times 10^{-8} \text{ cm s}^{-2}$  and directed toward the Sun for both spacecrafts. Pioneers 10 and 11 have very different trajectories out of the ecliptic and this implies that, if the cause of the acceleration is external to the spacecraft, the force is isotropic.

Proposals to explain the anomalous acceleration can mostly be grouped according to whether they are internal (*e.g.*, gas leaks, asymmetric emission of radiation) or external to the spacecraft. In the latter case some models hypothesize new fundamental physics (*e.g.* local cosmological effects) or novel applications of known phenomena (*e.g.* a dark matter halo around the Sun). One motivation for some of the external theories is the observation that  $a_p \approx Hc$ , where  $H$  is the Hubble constant and  $c$  is the speed of light. The relation is exact for  $H = 82 \text{ km s}^{-1}\text{Mpc}^{-1}$ . We refer the reader to Anderson *et al.* (2002) for a review and discussion of these numerous proposals. The proven accuracy of the ephemerides of Earth and Mars rule out the possibility that the anomalous acceleration both extends inside 1.5 AU and acts on these bodies of mass  $\sim 10^{27} \text{ g}$  (Anderson *et al.* 2002 ). Here we will present evidence that the anomalous acceleration also does not affect cometary bodies of mass  $\sim 10^{17} \text{ g}$  between 20-70 AU, the radial zone over which the unmodeled acceleration has been measured.

Upon passing through this radial zone, the anomalous acceleration would do positive

work on an inward bound comet of semimajor axis  $a$ , resulting in a gain of specific energy  $W_{Pioneer} = a_p \times \Delta r$ . For  $\Delta r = 50$  AU this is  $6.6 \times 10^7$  erg  $\text{g}^{-1}$ . In scaled binding energy units,  $x \equiv 10^6 \text{AU } a^{-1}$ , this corresponds to an energy shift of 15 units. We wish to consider whether a shift in binding energy of 15 units in all incoming new comets is consistent with the evidence that the galactic tide dominates in reducing Oort cloud comet perihelia to the observable region. This requires comparison of the mean original energy obtained from the Comet Catalog of Marsden and Williams (2001), with and without correction for the hypothetical shift, to that predicted by galactic tidal theory.

## 2 Comparison of the Catalog and predicted original energies

### 2.1 The Catalog original energies

New Oort cloud comets are those believed to be entering the inner planetary region for the first time, most likely because of galactic tidal perturbations. They are defined as having original semimajor axes  $\geq 10^4$  AU (or  $x \leq 100$ ). Determination of the original Keplerian energies given in the Comet Catalog is based on the observation of the comet's positions and times which give the osculating orbital elements near perihelion. The osculating orbit is then integrated back in time taking into account planetary perturbations to obtain the original Keplerian energy prior to entering the planetary region. In this analysis we will refer to the original binding energies obtained from the Comet Catalog as  $x_{cat}$  to distinguish them from the true original energies ( $x_{true}$ ) one would infer in the

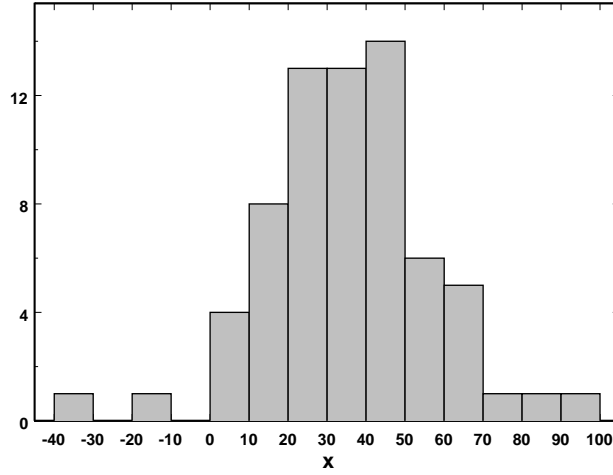


Figure 1: The distribution in binding energy of the 69 IA new comets used in the analysis.

presence of the anomalous acceleration, if real. The effects of outgassing are not included in the determination of the original energy quoted in the Catalog. The uncertainties in  $x_{cat}$  are discussed below.

For our data analysis we use the subset of new comets with the best determined orbits and Catalog energies prior to entering the planetary region. These comets are listed as quality class IA in the Catalog. The mean uncertainty in class IA comets is  $\pm 5$  compared to  $\pm 12$  for class IB (Kresak 1992). The split comet C/1996 J1-A(B) with catalog energy  $x_{cat} = -510(0)$  is included as a single comet with  $x_{cat} = 0$ . The distribution in original energies of these 69 IA comets is shown in Fig. 1. The mean original energy of these comets is  $\bar{x}_{cat} = 34$ . The standard deviation of the comet energies is

$$\sigma_{cat} = \sqrt{\frac{\sum_{i=1}^N (x_{cat}^i - \bar{x}_{cat})^2}{N - 1}} = 24$$

and the standard error of the mean is

$$\Delta \bar{x}_{cat} = \frac{\sigma_{cat}}{\sqrt{N}} = 3.$$

## 2.2 The true original energies

The specific work done on a comet as it falls from aphelion to its osculating (observed) location due to all forces other than the solar gravitational force is

$$W_{planets} + W_{outgas} + W_{Pioneer} = \Delta E = E_{osc} - E_{true} = \frac{GM_{\odot}}{2 \times 10^6 \text{AU}} (-x_{osc} + x_{true})$$

or, since

$$W_{planets} = \frac{GM_{\odot}}{2 \times 10^6 \text{AU}} (-x_{osc} + x_{cat})$$

then

$$x_{true} = x_{cat} + \frac{2 \times 10^6 \text{AU}}{GM_{\odot}} (W_{outgas} + W_{Pioneer}).$$

Neglecting outgassing, if the anomalous acceleration acts on the comets the true distribution of binding energies is systematically shifted upwards,

$$x_{true} = x_{cat} + 15 \frac{\Delta r}{50 \text{AU}}.$$

We argue below the effects of outgassing on this population of comets are likely to be random and not systematic.

## 2.3 The predicted original energies

The dominance of the galactic disk tide in reducing the perihelia of new Oort cloud comets into the observable region of the Solar System is well established (Heisler and

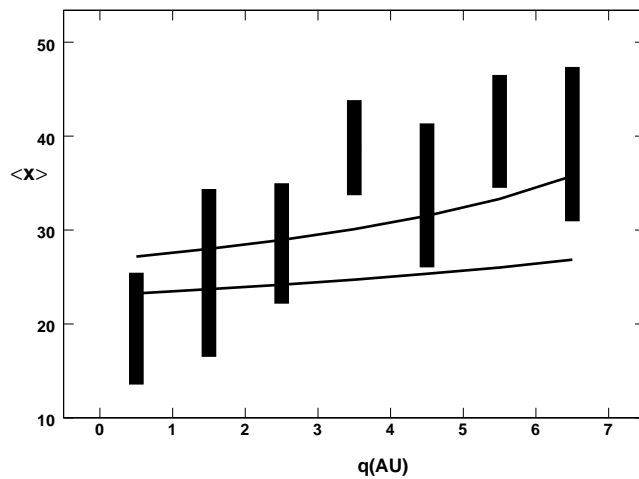


Figure 2: The vertical bars show the mean catalog binding energy and the associated 1 sigma standard error of the mean as a function of  $q$  averaged over intervals of  $\Delta q = 1$  AU. The data in bin 7 includes all comets with perihelia greater than 6. The theoretical curves shown bracket the range of parameters used in the galactic tide model. The upper curve is for  $\rho_o = 0.12 M_\odot \text{ pc}^{-3}$  and  $q_{lc} = 10$  AU, and the lower curve is for  $\rho_o = 0.08 M_\odot \text{ pc}^{-3}$  and  $q_{lc} = 15$  AU.



Tremaine 1986; Delsemme 1987; Matese and Whitman 1989; Wiegert and Tremaine 1999, Matese and Lissauer 2003). For example, the tidal torque is ineffective for comets with aphelia that lie near the galactic equator and poles, consistent with observations. Known observational selection effects cannot explain this pattern (Matese *et al.*, 1998). Tidal theory also predicts that the sign of the rate of change of the osculating specific angular momentum,  $\dot{H}$ , is preferentially negative if the galactic tide dominates in reducing the perihelion distance of comets, making them observable (Matese *et al.*, 1989; 1999). Calculation of this sign for the 69 IA comets in the 2001 Catalog shows that 47 out of 69 are negative. The probability of 47 or more out of 69 occurring by chance is 0.0018. In addition there are several other tidally predicted correlations which are found to exist with varying degrees of statistical confidence (Matese and Whitman 1989; Matese and Lissauer 2003).

The energy of comets that can be made visible by the galactic tide is unaffected by the assumed anomalous acceleration. Since the anomalous acceleration is radial it cannot change the comet's angular momentum and hence its perihelion. The strength of the tidal interaction is proportional to  $\Delta\sqrt{q} \propto \rho_o x^{-\frac{7}{2}}$ , where  $\rho_o$  is the local galactic density (Matese and Whitman 1989). Therefore, if the true distribution in binding energies is shifted upward significantly, the tide will be unable to make comets observable and its characteristic imprint will be absent in the data.

The uncertainty in mean energy in the theoretical prediction is that associated with the model parameters. The parameters are: (1)  $\rho_o$  (assumed azimuthally symmetric) ; (2) the radius of the loss cylinder  $q_{lc}$ , which is the average heliocentric distance inside of which comets will have their orbits perturbed sufficiently so that they are unlikely

to return to the new Oort cloud population, i.e. the rms energy change will be greater than 100; (3) the *in situ* distribution of Oort cloud comet energies.

In Fig. 2 we show theoretically predicted mean energies  $\bar{x}$  as a function of perihelion distance. The model parameters shown cover the ranges:  $\rho_o = 0.08 - 0.12 M_\odot \text{ pc}^{-3}$  (Holmberg and Flynn 2000) and  $q_{lc} = 10 - 15 \text{ AU}$  (*e.g.* Heisler and Tremaine 1986). The *in situ* distribution used is modeled after that of Duncan *et al.* (1987). Fig. 2 yields a theoretical mean energy (averaged over  $q$ ) of  $\approx 29$  with a range of  $\pm 3$ .

The theoretically predicted mean energy and its uncertainty ( $29 \pm 3$ ) is reasonably consistent with the Catalog value ( $34 \pm 3$ ). The small discrepancy in the unshifted energy, if real, may be due to a small impulse contribution (Matese and Lissauer 2002; 2003) which would tend to increase the predicted mean binding energy of comets made observable. Alternatively, the simplistic modeling of the loss cylinder barrier as a single distance, independent of comet inclination, may lead to errors at the larger  $q$  values where the discrepancy is greatest. Correcting for the anomalous acceleration would shift the Catalog mean binding energy to a true value of  $49 \pm 4$ , which is no longer consistent with the dominance of the galactic tide. A comet with a Catalog binding energy of 34 and a true binding energy of 49 would have  $\Delta\sqrt{q}$  reduced by a factor of 3, making the tide incapable of moving that comet's perihelion from beyond the loss cylinder barrier into the observable region.

### 3 Discussion

The Comet Catalog itself does not include observational uncertainties  $(\delta x)_{obs}$  in the original energies. For the class IA comets Kresak (1992) gives  $(\delta x)_{obs} = \pm 5$ . This error includes both the uncertainty in  $x_{osc}$  and the smaller errors associated with the planetary perturbations  $W_{planets}$ . The actual uncertainties will be larger due to the effects of outgassing. The Catalog energy standard deviation,  $\sigma_{cat} = 24$ , the theoretical population standard deviation  $\sigma_{th} = 8$  (Matese and Lissauer 2002), and the observational uncertainties satisfy

$$\sigma_{cat}^2 \approx \sigma_{th}^2 + (\delta x)_{obs}^2 + (\delta x)_{outgas}^2 .$$

Inserting the above values gives

$$(24)^2 \approx 8^2 + 5^2 + (\delta x)_{outgas}^2 .$$

This suggests that the original energy uncertainty due to outgassing is approximately 4 times the observational energy uncertainty for this data set. The disagreement between  $\sigma_{cat}$  and  $\sigma_{th}$  is not evidence that the galactic tide does not dominate. The wings of the Catalog distribution in Fig. 1 (negative  $x$ 's and  $x$ 's between  $\approx 50$  and 100) cannot be explained by tidal theory. We attribute this primarily to the expected uncertainties in energies, dominated by outgassing, whose effects are ignored in calculating the Catalog original energies. An additional smaller blurring of the energy distribution could be due to the estimated 10% of "new" comets which have in reality previously passed through the inner planetary region (Fernandez 1981).

For our high quality data set the effects of outgassing are nearly random as indicated by the symmetry of the distribution in Fig. 1. Therefore outgassing would not hide the

effects of the hypothetical anomalous acceleration if it acted on comets. A systematic tendency for outgassing to reduce  $x$  and make catalog energies more hyperbolic is most evident at small  $q$ , as shown in Marsden *et al.* (1978), and occurs predominantly in lower quality class orbits.

## 4 Summary

New comets listed in the Catalog of Marsden and Williams (2001) have orbital elements that show that they are predominantly made observable by the galactic tide. If they were truly bound as tightly as implied by the assumed externally caused Pioneer effect acting over 20-70 AU then the tide would be incapable of making these comets observable.

Correcting the Catalog mean original energy  $34\pm 3$  for the hypothetical Pioneer shift we would obtain a true mean original energy of  $49\pm 4$ . It is highly unlikely that this shifted mean energy is compatible with the mean tidally predicted energy of  $29\pm 3$ . We conclude that the Pioneer 10 anomalous acceleration is not due to an external isotropic force that acts on cometary mass bodies.

## ACKNOWLEDGMENTS

DPW acknowledges support of a University of Louisiana at Lafayette Summer Sabbatical. JJM acknowledges the support of a NASA-ASEE Faculty Fellowship at Ames. We thank Dr. J. Fernandez and an anonymous reviewer for valuable suggestions.

## References

- [Anderson *et al.* 2002] Anderson, J. D., P. A. Laing, E. L. Lau, A. S. Lin, M. M. Nieto, and S. G. Turyshev 2002. Study of the anomalous acceleration of Pioneer 10 and 11 *Phys. Rev. D* **65**, 082004.
- [Delsemme 1987] Delsemme, A. H. Galactic tides affect the Oort cloud: an observational confirmation. *Astron. Astrophys.* **187**, 913-918.
- [Ducan *et al.* 1987] Duncan, M., T. Quinn, and S. Tremaine 1987. The formation and extent of the Solar System Oort cloud. *Astron. J.* **94**, 1330-1338.
- [Fernandez1981] Fernandez, J. New and evolved comets in the Solar System. *Astron. Astrophys.* **96**, 26-35.
- [Heisler and Tremaine 1986] Heisler, J. and S. Tremaine 1986. Influence of the galactic tidal field on the Oort cloud. *Icarus* **65**, 13-26.
- [Holmberg and Flynn 2000] Holmberg, J. and C. Flynn 2000. The local density of matter mapped by HIPPARCOS. *Mon. Not. R. Astron. Soc.* **313**, 209-216.
- [Kresák 1992] Kresák, L. 1992. Are there any comets coming from interstellar space? *Astron. Astrophys.*, **259**, 682-691.
- [Marsden *et al.* 1978] Marsden, B. G., Z. Sekanina, and E. Everhart 1978. New osculating orbits for 110 comets and analysis of original orbits for 200 comets. *Astron. J.* **83**, 64-71.

- [Marsden and Williams 2001] Marsden, B. G. and G. V. Williams 2001. Catalog of cometary orbits, 14th edition. Smithsonian Astrophysical Observatory, Cambridge, MA.
- [Matese and Whitman 1989] Matese, J. J., and P. G. Whitman 1989. The galactic disk tidal field and the nonrandom distribution of observed Oort cloud comets. *Icarus* **82** 389-401.
- [Matese *et al.* 1998] Matese, J. J., P. G. Whitman and D. P. Whitmire 1998. Oort cloud comet perihelion asymmetries: Galactic tide, shower or observational bias? *Cel. Mech. Dyn. Astron.* **69**, 77-88.
- [Matese *et al.* 1999] Matese, J. J., P. G. Whitman and D. P. Whitmire. Cometary evidence of a massive body in the outer Oort cloud. *Icarus* **141**, 354-356.
- [Matese and Lissauer 2002] Matese, J. J. and J. J. Lissauer 2002. Characteristics and frequency of weak stellar impulses of the Oort cloud. *Icarus* **157**, 228-240.
- [Matese and Lissauer 2003] Matese, J. J. and J. J. Lissauer 2003. Continuing evidence for an impulsive component of Oort cloud cometary flux. *Proc. of Asteroids, Comets, Meteors (ACM 2002)* (ESA-SP-500), 309-314.
- [Wiegert and Tremaine 1999] Wiegert, P., and S. Tremaine 1999. The evolution of long-period comets. *Icarus* **137**, 84-122.