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DISCRETE OUT OF CONTINUOUS: DYNAMICS OF PHASE PATTERNS IN CONTINUA

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Abstract. A new paradigm is discussed in which the discrete facet of physical phenomenology is related to the existence of localized wave solutions (*solitons*) of a underlying *metacontinuum* (unified field, physical vacuum, etc.) while the action at a distance is a manifestation of the internal stresses in *metacontinuum*. The latter is considered as thin elastic 3D layer (a shell) in the 4D geometrical space. Maxwell's equations are recovered as corollaries from the equations of the motion in the middle surface. The equation for the amplitude of transverse flexural deformation appears to be a fourth order dispersive equation similar to the one derived by Schrödinger. The localized flexural (curvature) waves propagating *over* the surface of *metacontinuum* are interpreted as particles. A possible unification of the gravitation, electromagnetic phenomena and wave mechanics is demonstrated.

Introduction

The apparent duality between the point particles and field(s) underlies the foundation of modern physics. The Aristotelian tradition was connected with the notion of continuum but with the advance of the experimental physics the attention gradually shifted to the concept of point particles (corpuscles) moving freely in a void geometrical space. This new concept was heralded by Newton in his first two laws and it gave rise of a fruitful quantitative description of the motion of bodies known nowadays as *Newtonian Mechanics*. Yet it was Newton himself who admitted also an action at a distance when formulating the law of gravitation. Thus from the very beginning was planted the seed of dualism.

It is hard to imagine an action at a distance without some kind of a carrier. The nineteen-century tradition was always connected with some mechanical construct. In Cauchy's and Hamilton's vision it was a lattice whose continuum ap-

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roximation yielded the elastic-body model.[2] Paying tribute to the metaphysical radition, all of the models for the presumably absolute continuous media underlying he physical world were called "aether, " although some of them (the different elastic 1odels, for instance) were quite different from an "aetherial fluid" (where the coinage omes from). In Maxwell's imagination it was a medium with internal degrees of freeom. McCullagh and Sommerfeld quantified this idea as an elastic body with special otational elasticity (see [1] for exhaustive review of these theories).

The downfall of the concept of aether began with the unsatisfactory results f Cauchy's "volatile aether" (elastic body with vanishing dilational modulus). After ord Kelvin came up with the model of fluid aether and its vortex theory of matter, he coinage "aether" assumed almost exclusively fluid meaning. Then the question f entrainment of aether (aether-drift) was posed and the nil result of Michelson nd Morley experiment blew down the whole edifice of aether theories. However, the otion of a material carrier of the long-distance interactions could not be dismissed so asily and the conceptual vacuum was filled by the concept of what is called nowadays physical vacuum" which possessed all the properties of the disgraced aether (e.g., ction at a distance and giving birth to particles), but was deliberately exempted om the obligation to be checked for aether-drift effect. It was then advertised as a hing in itself not connected to any "primitive mechanistism." The same conceptual oad carries the coinage "field."

On a new level, the yearning for continuum description resurrected in the lamiltonian-Schrödinger wave mechanics. From the far side de Broglie quantified he wave properties of the particles. At that moment it was not clear how the two acets of the same object—the material particle—can peacefully coexist and it led to he concept that some kind of "pilot" wave was associated with a "point particle" epresenting the probability to find the latter at certain spatial position. Thus the ichotomy was deeply implanted in the physical thought and is still the dominant eneral attitude in the modern physics. As a result there has appeared a rather ntricate web of coinages (*pseudo, quasi*, etc.) when certain phenomenological fact ertaining to the one facet was being explained in the terminology of the dual facet.

In our point of view, the *field* (*physical vacuum, aether*) can only be undertood from the point of view of a material continuum where the *internal stresses are he transmitter of the long-range interactions*. In order to distinguish it from the 1echanical continuous media (bodies, liquids, gases, etc.), we call the continuum1echanics model of the unified field *metacontinuum* in the sense that it is beyond *meta*) the observable phenomena and is their progenitor. Returning to the conept of absolute continuum will be senseless without some new paradigm concerning he "point matter." It is not possible to imagine point particles pushing their way hrough infinitely stiff, virtually incompressible elastic body. Even if they could, the isturbances would be the predominant effect which has not been observed in any xperiment. So the new paradigm has to deal with inventing the concept of a particle

[2]It is curious to mention here that Cauchy came up with the elasticity theory because of his ttempts to constitute the aether.

which does not disturb the underlying continuum during its motion. The new concept we implant on the old tree of the absolute medium is that a particle is not something foreign to the field and immersed in the latter. Rather it is a localized wave of the field which *propagates over* the field (as a phase pattern) and is not moving *through* the field. Then we have to find phase patterns that are permanent enough in order to qualify for particles. Such individualized waves are well known from the soliton theory.

J. Scott Russell who was a contemporary of the golden age of the aether theories provided [2,3] the experimental evidence that a *nonlinear* field (2D surface of ideal fluid) can exhibit patterns of type of permanent waves with individualized comportment. The theoretical explanation was found by Boussinesq [4] and Lord Rayleigh [5] and it appeared that the balance between nonlinearity and dispersion is which sustains the long-living phase pattern. Zabusky and Kruskal [6] showed numerically that the solitary waves of KdV equation (close relative to Boussinesq equation) preserve their shapes and energies upon collisions and called them *solitons* to stress the particle-like behaviour ("quasi-particles").

Thus the stage for the unification was set up long time ago though it was not recognized as such. The only remaining question is which is the field (*metacontinuum*) whose solitary-wave phase patterns are the particles. A valid candidate for the luminiferous field is the elastic medium because it gives a good quantitative prediction for the shear-wave phenomena (light). As it is shown in [7] and briefly outlined here, the Maxwell equations are corollary from the linearized governing equations of the metacontinuum provided that the electric and magnetic filed are properly understood as manifestation of the "meta" internal stresses. The main difference from Cauchy's elastic aether is that we consider the opposite limiting case: an elastic continuum with infinitely large dilational modulus (virtually incompressible elastic medium).

There are other candidates for the field, e.g., the *sine*-Gordon equation considered in [8,9]. It is believed to be the meson field possessing solutions of type of "quasi-particles." Oddly enough, those localized waves are of type of fronts (kinks) which though mathematically well localized hardly fit into the common-sense picture of a particle. Yet the spatial derivatives of the solution did have the expected shape and the kinks qualify for *solitons* because the energy is conserved during the collisions. Nowadays there are many other "fields" and new are being introduced *ad hoc* for explaining one or another of the hundreds of elementary particles already known.

Here we stay firmly on the position that there is only one *metacontinuum* and the different fields are different manifestation of its internal stresses and strains. The "organization" of the matter around some terminology like "particles," "charges," "spins" is simply our way to simplify and thus to comprehend the complexity and richness of the interactions of localized phase patterns (solitons). For this reason we prefer to call the present attempt for an unified field theory "Soliton Paradigm." And the most ambitious goal is to derive the electromagnetic phenomena, the gravitation and wave mechanics from a single mechanical model of a metacontinuum.

Clearly, if one is to unify the wave function with the electromagnetic phenomena one has to consider more than three spatial dimensions. In addition, the thickness

ı the fourth dimension of the "material world" must be so small that it cannot be erceived by our senses. This idea was put forward by Hinton [10] but on qualitative asis only. Kalutza [11] and Klein [12] also considered additional "underdeveloped" imensions, but they added a vector field to the Einstein equation while we add a *calar* field to the Maxwell equations, the latter appearing here in the guise of a vector quation for shear waves in elastic continuum.

Carrying on with this idea we ask the question: "What kind of manifestation ; to be expected from the fact that the material world is a thin 3D layer in the 4D geometrical space?" Naturally, the presence of "underdeveloped" dimension(s) should esult into some new observables in addition to those that were sufficient to make self-consistent picture in the Maxwellian framework. Depending on the topology f the *metacontinuum* there may be (or may not be) a number of these manifestaions, connected with different spins in multi-dimensional space. The existence of at east one additional variable is inevitable, namely the amplitude of deflection of the D layer in direction of the fourth dimension. In Section 2 we derive an equation "master equation of wave mechanics") governing the said amplitude for geometrially nonlinear very thin (virtually $N^{\mathcal{D}}$) elastic layers in $N+1$ dimensions. We call his kind of mechanical construct *gossamer*. Naturally, this equation is not a followp from the governing equations in N dimensions, just like in the real world the chrödinger equation is not a corollary form the Maxwell equations.

The rest of the paper is devoted to arranging a cosmological picture of the olitary waves—phase patterns. In many instances they appear to be quite similar to hat the long-longed unified theory is expected to bring.

. Constituting a Metacontinuum

For small velocities the Lagrangian and Eulerian descriptions of a continuum oincide and for the displacements $\boldsymbol{u}$ of a Hookean elastic medium one has the linear ector wave equation

$$\mu_0 \frac{\partial \boldsymbol{A}}{\partial t} \equiv \mu_0 \frac{\partial^2 \boldsymbol{u}}{\partial t^2} = \eta \Delta \boldsymbol{u} + (\lambda+\eta)\nabla(\nabla\cdot\boldsymbol{u}) \equiv -\eta\nabla\times\nabla\times\boldsymbol{u} + (\lambda+2\eta)\nabla(\nabla\cdot\boldsymbol{u}) \,, \quad (1.1)$$

here $\boldsymbol{u}$, $\boldsymbol{A}$ are the displacement and velocity vectors; η, λ are Lamé's elasticity oefficients and μ_0 is the density in material (Lagrangian) coordinates. Note that we oncern ourselves for the time being only with a metacontinuum of constant elastic oefficients η, λ and density μ_0.

The full set of physical motions governed by (1.1) includes shear and compresion/dilation as well. The former are controlled by the shear Lamé coefficient η, while he latter—by the dilational (second) Lamé coefficient λ, and more specifically by the um $(\lambda + 2\eta)$. The phase speeds of propagation of the respective small disturbances re

$$c = \left(\frac{\eta}{\mu_0}\right)^{\frac{1}{2}} , \quad c_s = \left(\frac{2\eta+\lambda}{\mu_0}\right)^{\frac{1}{2}} , \quad \delta = \frac{\eta}{(2\eta+\lambda)} \,. \quad (1.2)$$

Here c and c_s are the speeds of shear waves (*light*) and compression waves (*sound*), respectively. In the case of very large dilational modulus, the speed of sound is much greater than the speed of light and $\delta \ll 1$. This is the opposite limiting case than the "volatile aether" of Cauchy with $\eta \gg \lambda$. Then Eq. (1.1) is recast as follows

$$\delta\left(c^{-2}\frac{\partial^2 \boldsymbol{u}}{\partial t^2} + \nabla\times\nabla\times\boldsymbol{u}\right) = \nabla(\nabla\cdot\boldsymbol{u}), \tag{1.3}$$

and displacement $\boldsymbol{u}$ can be developed into power asymptotic series with respect to δ

$$\boldsymbol{u} = \boldsymbol{u}_0 + \delta\boldsymbol{u}_1 + \cdots . \tag{1.4}$$

Introducing (1.4) into (1.3) and combining the terms with like powers we obtain for the first two terms

$$\nabla(\nabla\cdot\boldsymbol{u}_0) = 0, \tag{1.5}$$

$$c^{-2}\frac{\partial^2 \boldsymbol{u}_0}{\partial t^2} + \nabla\times\nabla\times\boldsymbol{u}_0 = \nabla(\nabla\cdot\boldsymbol{u}_1). \tag{1.6}$$

From (1.5) one can deduce

$$\nabla\cdot\boldsymbol{u}_0 = \text{const}, \quad \text{or} \quad \nabla\cdot\boldsymbol{A}_0 = 0, \tag{1.7}$$

which is also a linear approximation to incompressibility condition for a continuum. In the general model of nonlinear elasticity with finite deformations the incompressibility condition is imposed on the Jacobian of transformation from material to geometrical variables, but in the first-order approximation in δ the Eq. (1.7) holds true.

From here on we omit the index '0' without fear of confusion. We denote formally the term $(\lambda + 2\eta)\nabla\cdot\boldsymbol{u}_1$ by $(-\varphi)$ and recast (1.6) as dimensional form of linearized Cauchy balance, namely

$$\mu_0\frac{\partial\boldsymbol{A}}{\partial t} = -\nabla\varphi + \nabla\cdot\boldsymbol{\tau}, \tag{1.8}$$

where $\boldsymbol{\tau}$ is the deviator stress tensor for which the following relation is obtained from the constitutive relation (the Hooke law) for elastic body, namely

$$\boldsymbol{\tau} = \eta(\nabla\boldsymbol{u} + \nabla\boldsymbol{u}^T) - 2\eta(\nabla\cdot\boldsymbol{u})\mathbf{I}, \tag{1.9}$$

where $\mathbf{I}$ stands for the unit tensor. For the divergence of $\boldsymbol{\tau}$ one has

$$\nabla\cdot\boldsymbol{\tau} = -\eta\nabla\times(\nabla\times\boldsymbol{u}). \tag{1.10}$$

What is essential for the unification is that the linearized equations of elastic continuum admit what we call Maxwell form. The derivations here are not to be confused with McCullagh's model of pseudo–elastic continuum (see [1,13] for references and further developments) with restoring couples by means of which he tried

o explain the unusual shape of Maxwell's equations apparently not fitting into the icture of continuum mechanics. Let us introduce the vector field

$$\boldsymbol{E} \stackrel{def}{=} -\nabla\cdot\boldsymbol{\tau} \equiv \eta\nabla\times(\nabla\times\boldsymbol{u}). \tag{1.11}$$

o which the action of the purely shear part of internal stresses is reduced. It has the ieaning of a point-wise distributed body force and we shall call it "electric force." ı terms of $\boldsymbol{E}$, the linearized system (1.8) yields

$$\boldsymbol{E} = -\frac{\partial\boldsymbol{A}}{\partial t} - \nabla\varphi, \tag{1.12}$$

ıvolving the well known vector and scalar potentials $\boldsymbol{A}$ and φ. In the framework f the present approach, however, these potentials are not non-physical quantities ıtroduced merely for convenience, but rather they appear to be the most natural ariables: velocity and pressure of metacontinuum. Taking the *curl* of (1.12) one btains

$$\nabla\times\boldsymbol{E} = -\frac{\partial\boldsymbol{B}}{\partial t} \tag{1.13}$$

hich is nothing else but the first of Maxwell's equations (the Faraday law) provided hat a "magnetic induction" $\boldsymbol{B}$ is defined as

$$\boldsymbol{B} = \mu_0\nabla\times\boldsymbol{A} = \mu_0\boldsymbol{H} \quad \boldsymbol{H} \stackrel{def}{=} \nabla\times\boldsymbol{A}, \tag{1.14}$$

here $\boldsymbol{H}$ is called "magnetic field." From Eq. (1.11) one obtains

$$\frac{1}{\eta}\frac{\partial\boldsymbol{E}}{\partial t} = \nabla\times(\nabla\times\frac{\partial\boldsymbol{u}}{\partial t}) \equiv \nabla\times\boldsymbol{H}. \tag{1.15}$$

The last equation is precisely the "second Maxwell equation" provided that he shear elastic modulus of metacontinuum is interpreted as the inverse of electric ermittivity $\eta = \varepsilon_0^{-1}$. This equation was postulated by Maxwell [14] as an improveıent over Ampere's law incorporating the so-called displacement current $\partial\boldsymbol{E}/\partial t$ in he Biot–Savart form. For the case of a void space, however, when no charges or urrents are present, the second Maxwell equation lives a life of its own and Amere's law plays merely heuristic role for its derivation. It is broadly accepted now hat the second Maxwell equation is verified by a number of experiments. Here we ave shown that it is also a corollary of the elastic rheology of the metacontinuum nd is responsible for the propagation of the shear stresses (action at a distance) in *ıetacontinuum.*

The two main equations of evolution of Maxwell's form have already been erived. The condition div $\boldsymbol{H} = 0$ (the third Maxwell equation) follows directly from he very definition of magnetic field. Similarly, taking divergence of Eq. (1.11), one nmediately obtains the fourth Maxwell equation div $\boldsymbol{E} = 0$. Thus we have shown that Maxwell's equations follow from the linearized governing equations of a Hookean elastic medium whose dilational modulus is much larger than the shear one.

2. A Curved Metacontinuum? The Gossamer

From the point of view of material continuum a curved medium can be considered only as being embedded into a geometrical space of at least one spatial dimension larger, e.g., the Maxwell luminiferous field can be considered as 3D material hypersurface in 4D geometrical space. It is hard to imagine a material surface which has no thickness. A 3D hypersurface is a mathematical abstraction for the material construct known as thin shell (or a membrane) when the so-called middle surface of the latter is considered. If the shell is flat then the electromagnetism will be the only phenomena to be observed in three dimensions. Let us examine now the consequences of the curvature and transverse (flexural) deflections of *metacontinuum*. The existence of another spatial dimension cannot be detected in 3D shear-waves (electromagnetic) experiments.

Following the previous sections we consider an elastic shell of a 4D material whose shear Lamé coefficient is much smaller than the dilational one (their ratio given by the small parameter δ). As usual in shell/plate/membrane theories the small parameter $\varepsilon = h/L$ is the most important one, where h is the thickness of the shell and L is the length-scale of the deformations of the middle surface. The thin-layer simplification apply when $\varepsilon \ll 1$. The problem is to find a correct way to reduce the 4D continuum mechanics to an effective 3D mechanics for the middle surface of the shell.

In the technological applications an additional assumption is tacitly made, namely that L is large ("shallow shells") and *shell* is called a "reasonably" thin elastic structure whose flexural deformation (deflections) are of unit order, the strains (gradients) are $O(L^{-1})$ small and the curvatures are of second-order in smallness. Here we relieve the limitation of large L and treat the case $1 \gg L \gg h$. Then the deflections must be small, the strains (gradients)—of unit-order, and curvatures—large. The standard shell theory is not sufficient for describing such an object which is geometrically strongly nonlinear. As shown in [7], when deriving the shell equations for this limiting case one has to acknowledge more terms responsible for the geometric nonlinearity. At the same time the material nonlinearity is not so important because of the vanishing thickness h. In order to distinguish them from the classical shallow shells we call this kind of very thin elastic layers undergoing very high strains *gossamers*.

2.1. Manifestation of Underdeveloped Dimensions

We summarize here the relevant items of derivation of *gossamer*'s theory. Wherever possible we keep the derivation general enough speaking about $N^{\mathcal{D}}$ layer in $(N+1)^{\mathcal{D}}$ space, but for the purposes of the present work $N = 3$ and $N+1 = 4$. The Cauchy form for $(N+1)^{\mathcal{D}}$ continuum reads

$$[\rho_* a^j - P^{ij}\|_i]\,\boldsymbol{g}_j = 0, \quad i,j = 1,\dots,N \,, \tag{2.1}$$

here ρ_* is material density of the $(N+1)^{\mathcal{D}}$ continuous media filling the $N^{\mathcal{D}}$ shell.)ne can call it "meta-density" in order to distinguish it from the 3D density already lentified as μ_0. It is important also to note that neither ρ_* nor μ_0 have anything ɔ do with the density of matter (number density of solitons). Here $\boldsymbol{g}_j$ are the orts f the curvilinear coordinate system; P^{ij} are the components of stress tensor; a^j are he components of the acceleration vector in the $(N+1)^{\mathcal{D}}$ space; $\|_i$ stands for the ovariant derivative in $(N+1)^{\mathcal{D}}$ space. We do not consider here $(N+1)^{\mathcal{D}}$ body forces.

Upon substituting the expressions for $\|_i$ in terms of $N^{\mathcal{D}}$ covariant derivatives (see [15,16] and the extensions in [7]), the Cauchy balance law (2.1) is recast into a ystem for the laminar components and a scalar equation for the $N+1$-st component. fter averaging (integrating) within the surfaces of the shell one gets to the second rder $O(\varepsilon^2)$ of approximation.

$$\rho_* \varphi^\alpha - \nabla_\beta \sigma^{\alpha\beta} = -2b^\alpha_\beta q^\beta - b^\beta_\beta q^\alpha \,, \tag{2.2}$$

$$\rho_* \varphi^{N+1} - \nabla_\beta q^\beta = b_{\beta\nu}\sigma^{\beta\nu} - c_{\beta\nu} m^{\beta\nu} + \rho_* \mathcal{F} \,, \tag{2.3}$$

where

$$q^\alpha = \frac{1}{h}\int P^{N+1,\alpha}\,ds \,, \quad \sigma^{\alpha\beta} = \frac{1}{h}\int P^{\alpha\beta}\,ds \,, \quad m^{\alpha\beta} = \frac{1}{h}\int s P^{\alpha\beta}\,ds,$$

$$\varphi^\alpha = \frac{1}{h}\int a^\alpha\,ds \,, \quad \psi^\alpha = \frac{1}{h}\int s a^\alpha\,ds \,, \quad \rho_* \mathcal{F} = \frac{1}{h}(P^{N+1,N+1}_{\text{up}} - P^{N+1,N+1}_{\text{lo}}) \,.$$

Iere the subscripts "up" and "lo" refer to the upper and lower shell surfaces $s = h/2$ nd $s = -h/2$, respectively. It is taken into account that no tractions are exerted upon he shell surfaces from the two adjacent $(N+1)^{\mathcal{D}}$ spaces. The integrals are understood s definite integrals in s between the shell surfaces while b and c are functions of the surface" coordinates only. The notation ∇ stands for a $N^{\mathcal{D}}$ covariant derivative ı which the coefficients that may possibly depend on the transverse coordinate are lready averaged.

The system (2.2), (2.3) is coupled by the "momentum–of–impulses" which can e derived from (2.1) to the same asymptotic order $O(\varepsilon^2)$ after multiplying it by s nd integrating across the shell, namely

$$-\rho_* \psi^\alpha + \nabla_\beta m^{\alpha\beta} = q^\alpha \,, \tag{2.4}$$

hich allows us to exclude the quantity q^α from the governing system and to obtain

$$\rho_* \varphi^\alpha = \nabla_\beta \sigma^{\alpha\beta} - 2b^\alpha_\beta \nabla_\nu m^{\beta\nu} - b^\beta_\beta \nabla_\nu m^{\alpha\nu}, \tag{2.5}$$

$$\rho_* \varphi^{N+1} = \nabla_\beta \nabla_\nu m^{\beta\nu} + b_{\beta\nu}\sigma^{\beta\nu} - c_{\beta\nu} m^{\beta\nu} + \rho_* \mathcal{F} - \rho_* \nabla_\alpha \psi^\alpha \,. \tag{2.6}$$

Here the notion of the geometrization of physics becomes transparent. If the bserver is confined to the $N^{\mathcal{D}}$ space of the middle surface he will appreciate the resence of the $N+1$-st dimension as additional terms in balance law (2.5), (2.6) which terms are not present in the Cauchy form for the $N^{\mathcal{D}}$ continuous media.[3] The said terms are proportional to the different curvature forms and this is the quantitative expression of Riemann–Clifford [17,18] idea that the physical laws are manifestation of deformations of the geometrical space.

2.2. Elastic Shell with Momentum Stresses

According to the Kirchhoff-Love hypothesis, the displacements u_α in the shell space are related to the $N^{\mathcal{D}}$ displacements $\tilde{u}_\alpha$ in the shell middle surface as follows

$$u_\alpha = \tilde{u}_\alpha - s\nabla_\alpha\zeta, \quad u_{N+1} = \zeta\,, \tag{2.7}$$

where ζ stands for the shape function of deformation (deflection) of the middle surface in direction of $(N+1)$-st dimension. This hypothesis is pertinent to the overall $o(\varepsilon)$ approximation since it amounts to neglecting terms proportional to s^2. Then we obtain

$$\varphi_\mu = \frac{\partial^2\tilde{u}_\mu}{\partial t^2}\,, \quad \nabla^\mu\psi_\mu = -\frac{h^2}{12}\frac{\partial^2\Delta\zeta}{\partial t^2}\,, \quad \varphi_{N+1} = \frac{\partial^2\zeta}{\partial t^2}\,. \tag{2.8}$$

The rotational inertia $\nabla^\mu\psi_\mu$ is of second order which justifies neglecting it in comparison with the transverse inertia φ_{N+1}.

In terms of coordinates that are measured along the arcs of the middle surface (precisely the material Lagrangian coordinates), the second fundamental form assumes the following simple form

$$b_{\alpha\beta} = \nabla_\alpha\nabla_\beta\zeta\ . \tag{2.9}$$

Note that for coordinates not coinciding with the arcs, the expression of the second fundamental form involves nonlinear terms.

It is time now to couple the Cauchy equations with constitutive relations. Unlike the Cauchy form, the full nonlinear constitutive relations cannot be derived in Eulerian framework. It goes beyond the framework of the present work to derive them in full detail, especially as far as the material nonlinearity for the laminar components is concerned. We resort here to *linear* constitutive relations in the form (see [19])

$$\sigma^{\alpha\beta} = (\lambda_* + \eta_*)g^{\alpha\beta}(\nabla_\nu\tilde{u}^\nu) + \eta_*\nabla^\beta\tilde{u}^\alpha\ , \tag{2.10}$$

$$m^{\alpha\beta} = -D_*\nabla^\alpha\nabla^\beta\zeta\,, \quad D_* = \frac{\eta_* h^2}{12}\,. \tag{2.11}$$

where D_* is called stiffness of shell. Here asterisks designate the $(N+1)^{\mathcal{D}}$ material properties. If the middle surface of *gossamer* is to behave as the Maxwell luminiferous field then the following relations must hold true

$$\frac{\lambda_*}{\rho_*} = \frac{\lambda}{\mu_0}\,, \quad \frac{\eta_*}{\rho_*} = \frac{\eta}{\mu_0}\,, \quad \frac{D_*}{\rho_*} = \frac{D}{\mu_0}\,.$$

[3] If one was lucky enough to have already the tensor analysis developed and the notion of continuous medium (with its Cauchy balance) already established.

Ipon introducing (2.10), (2.11) into Cauchy equations (2.5), (2.6) we get for the aminar components

$$\mu_0\frac{\partial^2\tilde{u}^\beta}{\partial t^2} = (\lambda+\eta)\nabla^\beta(\nabla_\nu\tilde{u}^\nu) + 2\eta\Delta\tilde{u}^\beta - D\,[\,2\nabla^\beta\,(\,\nabla\zeta\cdot\nabla(\Delta\zeta)\,) + (\Delta\zeta)\nabla^\beta\Delta\zeta\,]\ , \tag{2.12}$$

nd for the amplitude ζ of normal deflection in direction of $N+1$–st dimension

$$\mu_0\frac{\partial^2\zeta}{\partial t^2} = \mu_0\mathcal{F} + D\left[-\Delta\Delta\zeta + (\nabla_\beta\nabla_\delta\zeta)(\nabla^\beta\nabla_\mu\zeta)(\nabla^\mu\nabla^\delta\zeta)\right] + \frac{\mu_0}{\rho_*}\sigma^{\beta\alpha}(\nabla_\beta\nabla_\alpha\zeta)\,. \tag{2.13}$$

Iere we shall not consider the case of nontrivial tractions on the shell surfaces which equires additional assumptions about their asymptotic order. We keep, however, the erm responsible for the normal pressure (parameter F).

If we develop with respect to the powers of the small parameter δ in the same nanner as in the previous section, we get from (2.12) and (2.10) that

$$\nabla^\beta\nabla_\nu u^\nu = 0 + O(\delta) \implies \nabla_\nu u^\nu = \kappa_0 = \text{const} \implies \frac{\mu_0}{\rho_*}\sigma^{\alpha\beta} = \sigma_0 g^{\alpha\beta}\ ,\ \sigma_0 = \eta\kappa_0\ ,$$

vhere κ_0 is dimensionless divergence of the displacement field in the middle surface. Iote that $k_0 > 0$ means uniform dilation of the middle surface, while $k_0 < 0$ reflects he case of uniform compression. Thus, in the lowest asymptotic order of δ we are aced with uniform compression/dilation in the middle surface of *gossamer* and with onstant membrane stress σ acting in the middle surface. This allows to effectively ecouple the laminar deformations $\tilde{u}^\alpha$ from the deflection ζ even without requiring pecial relation between the scales for ζ and $\tilde{u}$. In its turn the term in (2.13) containing $^{\alpha\beta}$ becomes simply $\sigma_0\Delta\zeta$. Because of the large dilational modulus, the laminar eformations appear to be orthogonal to the transverse ones to the first asymptotic rder.

From here on the "tildes" denoting the laminar variables will be omitted withut fear of confusion.

.3. The Dispersive Equation of Wave Mechanics

The only term which looks unusual in (2.13) is the cubic nonlinear term. Howver, in dimension one the equation under consideration is exactly the cubic nonlinear 3oussinesq equation. In order to benefit from the vast knowledge accumulated for quations of type of Boussinesq, we replace in a paradigmatic fashion the cubic term 1 (2.13) by $(\Delta\zeta)^3$. It is beyond doubt that the qualitative behaviour of the solutions vill be quite similar. However, the extent to which they will be quantitatively close s well, remains to be verified. Then we arrive at

$$\mu_0\frac{\partial^2\zeta}{\partial t^2} = \mu_0\mathcal{F} + D\left[-\Delta\Delta\zeta + (\Delta\zeta)^3\right] + \sigma_0\Delta\zeta\,. \tag{2.14}$$

We render the last equation dimensionless by introducing the scales

$$\zeta = L\zeta' , \quad \boldsymbol{x} = L\boldsymbol{x}' , \quad t = \frac{L}{c_f}t' , \quad c_f = \sqrt{\frac{\sigma_0}{\mu_0}} \equiv c\sqrt{|k_0|} , \tag{2.15}$$

where c_f has dimension of velocity. Note that the scale for ζ and the length scale of the localized wave coincide (the length L), if one looks for commensurable effects (balance) of the nonlinearity and dispersion. This fits perfectly the original assumptions for *gossamer*: small deflections of order L, unit strains and large curvatures of order L^{-1}. Finally, the dimensionless form of the wave equation of *gossamer* reads

$$\frac{\partial^2 \zeta'}{\partial t^2} = \mathcal{F}' + \beta \left[-\Delta\Delta\zeta' + (\Delta\zeta')^3 \right] + \text{sign}[k_0]\Delta\zeta' , \tag{2.16}$$

where $\beta = D|\sigma_0|^{-1}L^{-2}$ is the dispersion parameter and $\mathcal{F}' = \mathcal{F}L\mu_0/|\sigma_0|$ is the dimensionless value of the normal load (hydrostatic pressure). From here on the primes denoting dimensionless variables are henceforth omitted without fear of confusion. Eq. (2.16) is our "master equation of wave mechanics."

Now β is the only intrinsic non-dimensional parameter and if it is significant, then without loosing the generality it may be set equal to one. This defines the length scale L of the particle-waves as

$$L \sim \sqrt{\frac{D}{\sigma_0}} \sim \frac{h}{\sqrt{|k_0|}} \quad \Longrightarrow \quad \varepsilon = \frac{h}{L} \simeq \sqrt{|k_0|} .$$

The last relation shows that the model is applicable only if the longitudinal compression of the *gossamer* is of order of the main small parameter, namely $\varepsilon \sim \sqrt{|k_0|} \ll 1$.

Some remarks are due concerning the "master wave equation" (2.16). Its linear part has the form originally proposed by Schrödinger (Eq. (4) from [20]). Naturally, Schrödinger himself mentioned the analogy with the plate equation and injected a remark in the cited paper. It was later on when the interpretation of the wave function as probability distribution has been introduced and the now standard form of the equation involving a complex wave function was introduced. In order to distinguish the originally derived equation from the "canonical" form we call the linear wave equation containing fourth-order dispersion Schrödinger's Schrödinger Equation (SSE). The derivation of Schrödinger was rather heuristic. Here we have arrived at qualitatively similar equation but bearing in mind a "palpable mechanical construct." The wave function now has a simple meaning: the deflection of *gossamer*'s middle surface in direction normal to it (alongside of the $(N+1)$-st dimension). These are *par excellence* curvature waves envisaged by Riemann [17] and Clifford [18].

3. The Cosmos of Localized Structures in Gossamer (Dynamics of Patterns in the Metacontinuum)

3.1. A Model for Loading the Metacontinuum

A way to achieve a homogeneous and isotropic loading of *gossamer* is to con-

ider a large $N^{\mathcal{D}}$ bubble (hypersphere) subjected to hydrostatic pressure from the djacent $(N+1)^{\mathcal{D}}$ spaces. We call this hypersphere *Universe*. We assume that the ubble is compressed from the outside $(N+1)^{\mathcal{D}}$ (negative membrane stress in the iiddle surface). A discussion on the case when the model of *Universe* is an inflated ubble can be found in [7]. It is however, a kind of artificial since to get there the amiliar *sech*-solitons one has also to change the sign of the cubic term. Hence the inflated-bubble" model appears to have only heuristic significance. The motionless equilibrium) state is characterized by the balance between the membrane tension nd the hydrostatic compression creating a negative membrane tension in the middle urface, namely

$$\sigma_0 \frac{N}{R} = -\rho_* |\mathcal{F}| \quad \Longrightarrow \quad k_0 = -\frac{|\mathcal{F}|R}{Nc^2} < 0 .$$

Iere R is the radius of the bubble made dimensionless by the scale L, and N is the imension of the middle surface of the *gossamer* ($N = 3$ for the physical Universe).

It is clear that the dimensionless force $\mathcal{F}$ must be small enough (of order f the inverse of the dimensionless radius of Universe). We introduce the relative isplacement $\bar{\zeta} = \zeta - R$. Since we are interested in an Universe whose scale is much arger than the size L of its particles then the dimensionless radius R is extremely arge. Hence its derivatives are very small and can be neglected in the cubic term. 'or all purposes the shell can be treated as plane. Then Eq. (2.13) is reduced to the ollowing

$$\frac{\partial^2 \bar{\zeta}}{\partial t^2} = -\Delta\bar{\zeta} + \beta[(\Delta\bar{\zeta})^3 - \Delta\Delta\bar{\zeta}] , \tag{3.1}$$

nd $\beta = 1$ without loss of generality.

.2. Flexural Localized Structures—The Flexons

In the stationary case the localized waves under question have spherical symietry and for them the following dimensionless boundary value problem in infinite omain is posed

$$b - b^3 + \frac{1}{r^{N-1}} \frac{d}{dr} r^{N-1} \frac{d}{dr} b = 0, \quad b \to 0 \quad \text{for} \quad r \equiv |\mathbf{x}| \to \pm\infty , \tag{3.2}$$

'here $b = \Delta\zeta$ is the curvature of the nontrivial transverse elevations/depressions. 'he linearized version of Eq. (3.2) possesses along with the trivial solution a localized on-trivial one (the *sinc* function):

$$\zeta = ar^{-1} \sin r , \tag{3.3}$$

'here a is an arbitrary constant. This means that a linear bifurcation takes place 'hich makes our problem different from the classical soliton problems where a hard ıonlinear) bifurcation is at hand. The *sinc*-shape solution (3.3) has been just recently ıterpreted as a "single event" of the Schrödinger equation [21]. In fact the present 'ork justifies using the dispersive master equation (or Schrödinger equation) not as n equation for the probability density of a particle, but rather as a field equation (in the same fashion as *sine*-Gordon equation is used after [8]). It has to be pointed out that in our model we do not impose the shape of the potential as it is usually done in quantum mechanics.

We have solved (3.2) by means of an numerical algorithm based on the Method of Variational Imbedding (MVI) developed in [22] with application to homoclinics of Lorentz system (see also [23] for application to dissipative solitons). Because of the slow algebraic decay of the tails of the solution the interval has to be truncated at very large r. At the same time the grid has to be dense enough to allow resolving the oscillations. The results presented here employ grids with up to 40000 points and spacing 0.01. The solution turned out to be very sensitive to the mesh size and the magnitude of the "actual infinity," so some additional refinement could change the the amplitudes of solitons presented in Fig. 1 with couple of percents. Typically for nonlinear bifurcation problems, more than one nontrivial solution appear for the same values of the governing parameters. The question of the number of non-trivial solutions is of prime importance for the physical applications. Here we have found solutions with discrete set of amplitudes. This result suggests that for larger amplitudes the support of the main peak of the solution shrinks, i.e. the larger particles have shorter Compton wavelength. With the increase of the amplitude the behaviour of the solution in the origin $r = 0$ approaches r^{-1} which is a singular solution bringing a balance between the second order operator and cubic nonlinear term in (3.2). The physical significance of such kind of singular solutions is not yet understood in the framework of the proposed here paradigm and we did not go for larger amplitudes.

We identify the obtained here flexural solitary waves of the governing equations of gossamer as the *particles*. The fact that the metacontinuum is elastic does not necessarily mean elastic interaction of the particles (see the demonstration of this fact for the Boussinesq equation in [24]). As we show in what follows, the model under consideration is a conservative one, but to prove that our flexural localized waves live up to the definition of *solitons* is necessary either to find the two-soliton solution of (3.1) or to demonstrate the interactions of *flexons* numerically. We have done neither of these two things because the analytical techniques simply do not work for negative membrane tension. At the same time the 1D reduction of our model does not possess localized solutions. Numerical solution of a 2D problem with the above mentioned requirements for the grid size in each direction would have needed enormous computational resources. Until their solitonic properties are strictly established we will call the localized waves discovered here *flexons* which carries also a hint of their origin (flexural deformations or deflections).

As it will be seen later on, the amplitude of a *flexon* is directly related to the *mass* of particle. Naturally, a *flexon* with negative amplitude will be an *anti-particle*. The simplified Eq. (3.2) does not distinguish between particles and anti-particles. However, in the original model of *Universe* a slight difference between particles and anti-particles is to be expected due to to the curvature of the undisturbed bubble. Either the particles or the anti-particles will have a better chance to appear depending on which of them minimize the stored elastic energy of the shell.

One sees in Fig. 1 that the amplitudes (and hence—the masses) of flexons can

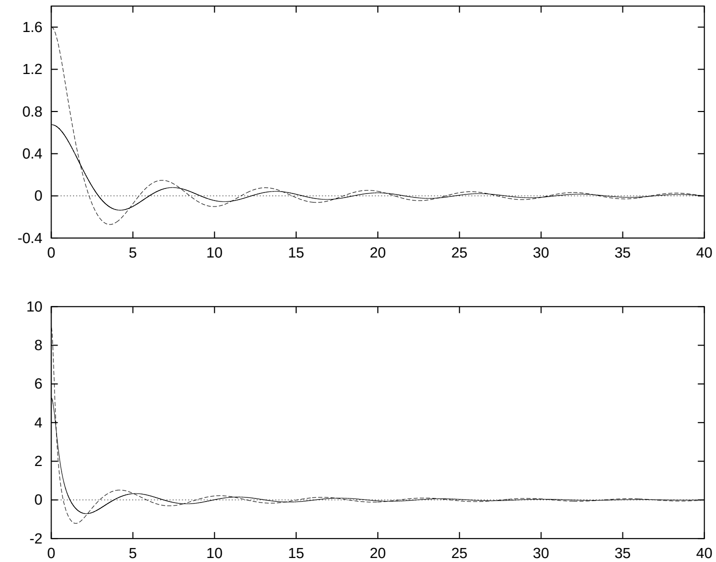

Fig. 1. Flexon solution with small amplitudes (up) and large amplitudes (down).

e quite different. To excite a bigger *flexon*, one has to invest greater energy as initial ondition of the iterations. Respectively, in the iterative process some of the bigger articles decay to smaller emitting part of their energy. In this instance, the situation ; qualitatively similar to relationship of nucleons and quarks.

.3. Localized Shear Waves in the Middle surface: Torsion Solitons (Twistons)

Although the way we load the gossamer is the simplest one (a uniform hyrostatic pressure), it turns out that a complex "cosmological" picture appears with ost of different localized waves. Bifurcation and symmetry breaking does not affect ıst the transverse deformation. Here we provide an example of what can happen ı the middle surface. Unfortunately the material nonlinearity has not yet been inorporated in the model and the linearized equations for laminar components remain ot closed due to the presence of the pressure-like term $\nabla\varphi$. For that reason the resented here solution has mainly qualitative heuristic bearing. By direct inspection ne finds that the vector

$$\boldsymbol{u} = \left\{ \frac{x(z^2 - y^2)}{(x^2 + y^2 + z^2)^{\frac{3}{2}}},\ \frac{y(x^2 - z^2)}{(x^2 + y^2 + z^2)^{\frac{3}{2}}},\ \frac{z(y^2 - x^2)}{(x^2 + y^2 + z^2)^{\frac{3}{2}}} \right\}$$

atisfies the "incompressibility condition" $\nabla \cdot \boldsymbol{u} = 0$. The vector lines corresponding ɔ this solution resemble much a vortex or a bundle. This deformation field creates an electric field given by

$$\boldsymbol{E} = \frac{12\boldsymbol{u}}{x^2+y^2+z^2}\,,$$

which is singular in the origin. It decays at infinity as r^{-2} which is in accord with the electromagnetic theory.

With the vortex-like solutions topological charges can be associated but it goes beyond the scope of present work to give the details, moreover that different topological charges can be defined and this still awaits its mechanical explanation. We call the localized solutions of vortex type *twistons* in order to distinguish them from the fluid vortices. The obvious symmetry of the linearized problem shows that the charge can be positive or negative. Depending on their charges, two *twistons* repel or attract each other (just as two vortices do). The latter means that the presence of a *twiston* can only be experienced by another *twiston*. The neutral particles (*flexons*) remain unaffected by the shear deformations in the middle surface of *gossamer* to the lowest asymptotic order. However, the $O(\delta)$ coupling between the laminar and transverse (flexural) deformations can cause a slight elevation (depression) of the *gossamer* surface in the region of localization of a *twiston*. In other words, the *twiston* has its own mass which can either be positive or negative and is much smaller than the mass of the *flexon* (the neutral particle). By analogy one can call it "mass of electron/positron." Massive charged particles can be produced when a *twiston* "nests on" a *flexon* and then the mass is the superposition of the amplitude of the neutron (*flexon*) and the amplitude of the flexural deformation associated with the *twiston* (positron or electron).

There is no much sense going on here with the details before the material nonlinearity of *metacontinuum* is established. In addition, the singularity of vortex solutions suggests that some higher-grade elasticity is to be admitted in order to change the behaviour of the solution for $r \to 0$, e.g., in the following fashion

$$\frac{\partial^2 \boldsymbol{u}}{\partial t^2} + \Phi(\nabla \boldsymbol{u}, \Delta \boldsymbol{u}) = \Delta \boldsymbol{u} - \nabla \varphi - \chi \Delta\Delta \boldsymbol{u}\,, \tag{3.4}$$

where Φ stands for the yet unspecified nonlinearity and χ is the dispersion coefficient.

Here a comment is due on the limiting speed of light. Clearly, the speed of light is not a limiting celerity for a *flexon*. A truly neutral *single* particle should not be affected by the speed of shear waves (light) as being orthogonal to shear deformation. However, as far as the charged particles are concerned, the speed of light is indeed the limiting speed because the *twistons* are solutions of the nonlinear wave equations for the laminar components of displacements. Hence an atom containing charged particles cannot exceed the speed of light since above it the charges (twistons) will be disjointed from the flexural "humps" that carry them.

3.4. Density Solitons

Alongside with the flexural and torsion solitary waves, one must expect also solitary waves connected with the compressibility of *metacontinuum*. Hence a quantitative numerical solution for the density solitons has not been attempted by us. How-

ver, some qualitative conclusions can be reached on the basis of the known properties f the compression waves in solids. First of all, the speed of these waves is limited y the speed of sound of the *metacontinuum* and the steepness of a density wave in-reases with its celerity. This means that the swifter the movement of the wave—the maller its spatial extent. On the other hand, the density solitons are expected to iteract almost insignificantly with the matter (the *flexons* and *twistons*) and in this istance they will resemble to a great extent the behaviour of the neutrino. This ualitative description allows us to consider the all pervading compression/dilation iotion (orthogonal to the matter) as quite similar to what the ancient School of Stoa alled *pneuma* (see, e.g., [25]).

.5. The Shell Membrane Tension or The Gravitation

According to the picture drawn here, the particles are localized elevations humps) of the gossamer surface. Due to the presence of a *flexon* of shape ζ situated t certain geometrical position, the material points will experience attractive force roportional to $|k_0|\nabla\zeta$. For a single *flexon* (see Fig. 1), this force will not be mono-one. The bare fact is that nobody has measured the gravitation force between two lementary particles and it is not clear whether the attractive force is monotone in he intra-atomic regions. In fact, the gravitation law is established only for bodies ensembles of *flexons*). So one has to consider only the flexural-deformation field as veraged over the different positions of the particles-flexons. It is to be expected that he total average is positive (attraction), since the positive humps of the *flexon* am-litude are larger than the negative. Taking the *sinc*-shape for qualitative purposes ne has approximately

$$\zeta = \sum_{\boldsymbol{x}_\alpha \in D_B} \frac{a_\alpha\, f|\boldsymbol{x}_\alpha - \boldsymbol{x}|}{|\boldsymbol{x}_\alpha - \boldsymbol{x}|} \approx \frac{g}{|\boldsymbol{x}|}, \quad g \sim \gamma \int f(|\boldsymbol{x}_\alpha|)\, d\boldsymbol{x}\,, \tag{3.5}$$

here D_B is the region occupied by the body. Here it is acknowledged that $|\boldsymbol{x}| \gg |\boldsymbol{x}_{\boldsymbol{\alpha}}|$ nd an assumption is made that the centers of particles are randomly distributed in he region of body with number density γ. Note that if the *flexons* were strictly *sinc*-haped, then the constant g would have been equal to zero for randomly dispersed toms (integral of *sine* is zero). For a *flexon* obtained here $g \neq 0$ because its amplitude 1 the origin is larger than of the *sinc*, so that after rescaling by $|\boldsymbol{x}|$ the *flexon* shape ives for the integral in (3.5) a positive quantity.

Thus the force that is experienced by a material point of the *gossamer* is roportional to $\nabla\zeta \sim G|\boldsymbol{x}|^{-2}$, $G = |k_0|g$ and both ingredients of G have extremely mall values. The exact value of the "gravitational constant" G can be specified only fter the averaging procedure is performed with the appropriate rigor. The membrane orce acts to "pull" the material points of the *gossamer* towards the center of the article system under consideration. Thus we arrive at Newton's inverse-square law f gravitation which is a manifestation of the fact that the shell is a 3D continuum.

In fact, the attraction between particles arises out of the disturbances they itroduce in the uniform membrane anti-tension acting in the shell, i.e. we discover a uantitatively reversed but philosophically identical picture to the concept of Mach that the gravitation is due to the interaction with the quiescent matter at the rim of Universe. Here apply the words of Maxwell from the end of Part IV of [14] "...that the presence of dense bodies influences the medium so as to diminish this energy whatever there is a resultant attraction." Indeed, the presence of humps over the gossamer surface influences the medium so as to diminish the stored energy and there arises a resultant attractive force.

In our model there is no place for gravitational waves because the membrane tension is negative hence the deflection waves would propagate infinitely fast were the dispersion not present. Bizarrely, the particles themselves can loosely be called "gravitational waves" since part of the forces constituting the particles is the gravitation (membrane anti-tension). In other words, a particle is a localized nonlinear wave sustained from the balance between the generating effect of the membrane anti-tension on the one hand, and the restraining effect of the cubic nonlinearity and dispersion on the other.

3.6. Dispersion and "Red Shift"

It is peculiar that the Boussinesq equation (3.4) possesses *localized* solutions (see [24,26]) that propagate with the characteristic speed (photons?) and undergo some aging ("red shift") in the sense that their support increases while the amplitude decreases. Far from the source, one cannot distinguish between the red-shifting due to dispersion or to a Doppler (effect if present). This means that if a dispersion is present then the "red shift" can be alternatively explained without the help of "Big-Bang" hypothesis.

3.7. Estimating the Constants of Metacontinuum

The density of *metacontinuum* is the magnetic permeability μ_0. It is well known that the whole electrodynamics can be built without specifying the dimension of μ_0. In our paradigm it is exactly to be expected that way, since μ_0 is a *meta* quantity. Its dimension is not needed anywhere in the model and the density of matter (being the number density of solitons) has nothing to do with it.

The shear Lamé coefficient η is the inverse of the electric permittivity ε^{-1}. The thickness of *gossamer* is proportional to Plank's constant $h \sim \hbar c^{-1}\mu_0^{-1}$. If there is a dispersion coefficient in the equations for laminar displacements it can be estimated from the Hubble constant. The dilational Lamé coefficient λ can be estimated only from an "acoustic" experiment in the metacontinuum.

4. Dynamics of Patterns in Metacontinuum

4.1. The Hamiltonian Formulation

Consider Eq. (3.1) in the domain D with boundary conditions for the wave-amplitude ζ

$$\zeta = \psi_0(\boldsymbol{x}), \quad \Delta\zeta = \psi_2(\boldsymbol{x}), \quad \text{for} \quad \mathbf{x} \in \partial D. \tag{4.1}$$

The trivial boundary conditions correspond to an isolated system of wave-particles. A Hamiltonian representation is readily derived upon multiplying (3.1) by $\Delta\zeta_t$ and

tegrating over the infinite domain.

$$H \equiv \frac{1}{2}\int_D \left[(\nabla\zeta_t)^2 - (\Delta\zeta)^2 + \frac{1}{2}\beta(\Delta\zeta)^4 + \beta(\nabla\Delta\zeta)^2\right] d^N\boldsymbol{x}. \tag{4.2}$$

or boundary conditions that do not depend on time, the total energy H is conserved $H/dt = 0$.

The energy functional (4.2) is not positive definite, but unlike the Boussinesq ave equation the solution will not blow up in finite time since the quartic nonlinarity dominates the quadratic term saturating the growth of solutions. Although he second-order term in (3.1) is of improper sign the equation is not unstable with espect to short wave lengths because of the presence of fourth spatial derivatives ith the proper sign.

The wave mass and wave momentum are defined as

$$M \equiv \int_{-\infty}^{\infty} \Delta\zeta \, \mathrm{d}^N\boldsymbol{x}, \quad \boldsymbol{P} \equiv \int_{-\infty}^{\infty} \zeta_t \nabla\Delta\zeta \, \mathrm{d}^N\boldsymbol{x}. \tag{4.3}$$

The concept of *pseudomomentum* in continuum mechanics was elaborated reently [27,28] especially in connection with the interpretation of localized waves as quasi-particles" and numerous featuring examples are presented in [29]. For this odel the meaning of *mass* is as "mass of curvature" which once again is in the vain f Riemann proposal. The *mass* is conserved if $\partial\zeta_{tt}/dn = 0$ at ∂D which is a natural equirement for an isolated system. For the *pseudomomentum* we have

$$\begin{aligned}\frac{d\boldsymbol{P}}{dt} &= \oint_{\partial D} \boldsymbol{n}\left[-\frac{1}{2}(\nabla\zeta_t)^2 - \frac{1}{2}(\Delta\zeta)^2 + \frac{\beta}{4}(\Delta\zeta)^4 + \frac{\beta}{2}(\nabla\Delta\zeta)^2\right] \mathrm{d}\boldsymbol{s} \\ &\quad - \oint_{\partial D} \beta(\nabla\Delta\zeta)\frac{\partial\Delta\zeta}{\partial n}\mathrm{d}\boldsymbol{s} = \mathcal{F}_{\mathrm{psu}}, \end{aligned} \tag{4.4}$$

here $\mathcal{F}_{\mathrm{psu}}$ is called *pseudoforce* and $\boldsymbol{n}$ is the outer normal to the region D. The *seudomomentum* is conserved if the *pseudoforce* is equal to zero. Note that even for n isolated system the *pseudomomentum* is not conserved if the localized patterns hit he boundary and rebound.

.2. From Metadynamics of Underlying Continuum to Dynamics of Centers of Localized Structures

The main significance of the Hamiltonian formulation is that it provides the eans to build up the dynamical model for the *discrete* phase objects (*solitons*). the shape of localized wave is known, the Hamiltonian dynamics for the discrete ystem of centers of "particles" can be derived from (3.1) with a good approximation rovided that they do not interact so strongly as to change appreciably their shapes. or two localized waves the wave amplitude can be decomposed as follows:

$$\zeta = F_1(\mathbf{x} - \mathbf{X}_1(t)) + F_2(\mathbf{x} - \mathbf{X}_2(t)) + F_{12}(\mathbf{x} - \mathbf{X}_1(t), \mathbf{x} - \mathbf{X}_2(t)). \tag{4.5}$$

ere F_i are the shape functions of the waves–particles and $\mathbf{X}_i(t)$ are the trajectories f their centers. When the particles are far from each other F_{12} is negligible. In fact

F_{12} must be considered only in the cross-section of the collision of two particles and we leave the problems connected with this case for future study.

Now the time derivative of each shape function can be expressed as follows

$$\frac{dF_i}{dt} = -\nabla F_i \cdot \frac{d\mathbf{X}_i}{dt} \tag{4.6}$$

and the discrete Hamiltonian contains quadratic forms of the velocities of centers

$$\sum_i \mathcal{A}_i \cdot \dot{\mathbf{X}}\dot{\mathbf{X}}\,, \tag{4.7}$$

where the matrix $\mathcal{A}_i$ is positive definite. Then (4.7) can be interpreted as definitive relation for the kinetic energy of the center of particle. Respectively, F_2 will contribute a term which depends on the relative position of particles and plays the role of potential of interaction. In absence of interactions, the Euler–Lagrange equations for discrete system of centers of *solitons* give $\ddot{\mathbf{X}} = 0$ and a *soliton* will propagate with constant phase velocity. Thus for isolated particles the first Newton law is recovered.

4.3. FitzGerald-Lorentz Contraction and Contraction of Flexons

The FitzGerald-Lorentz contraction (FG–L, for brevity) is a standard feature with the soliton solutions of generalized wave equations, e.g., SG, Boussinesq, etc. It is especially well seen for the *sine*-Gordon equation where the "quasi-particles" are called "relativistic"[4] meaning that their contraction is exactly proportional to the Lorentz factor. Naturally, the above discussed localized shear waves are subject precisely to the same factor of contraction. This holds true also for the length scales of interaction forces (e.g., the Coulomb force).

When a *flexon* propagates with a constant celerity V its measure in the direction of propagation must be shortened by the factor (see (2.15), (3.1))

$$\left(1 + V^2/c_f^2\right)^{-\frac{1}{2}}\,, \tag{4.8}$$

where c_f is the pseudo-velocity corresponding to the negative membrane tension. Respectively the amplitudes of *flexons* (related to their particle-wave masses) are increased roughly by the same factor. Since c_f is much smaller than the speed of light, the contraction of flexons must be felt for smaller phase velocities than the Lorentz contraction. On the other hand —contrary to the FG–L contraction—there is no singularity in the expression of *flexon* contraction (4.8). However, the new type of contraction is much less important for the contraction of the bodies since it only affects the scale of gravitational interaction which is anyway the weakest and is not important in defining the intra-atomic scales. Yet, it is quite possible that some small effect could be felt due to the flexural contraction. It will not be a surprise if the

[4]Another specimen from the glittering panoply of coinages which is misleading in this context since SG is a model of an absolute field

eak but persistent deviations found in [30] from the nil effect of the Michelson-Morley xperiment turn out to be the result of this additional contraction.

. A First-Order Experiment for Detecting the Doppler Effect

To use interferometry for verification of the Doppler effect was suggested by laxwell [31]. It was believed that discovering a Doppler effect will prove the existence f an absolute medium at rest. The experiment was implemented first by Michelson 32] and nil effect was observed. It was later on refined by Michelson and Morley [33] MM, for brevity) and the absence of the expected type of interference was confirmed ore decisively. In our opinion the nil effect of MM experiment cannot disprove the xistence of absolute medium because it seeks for a second-order effect in the small arameter $d = v_e c^{-1} \approx 10^{-4}$ (v_e stands for the velocity of Earth with respect to the uiescent medium). This was specially pointed out by Maxwell [31] way before the xperiment was performed. He proved that employing only a single ray with splitting nd reflections inevitably renders the sought effect of second order d^2 because the ght travels along a closed path. The only conclusion that can be drawn from the il effect is that in the medium where the light is being propagated there occurs an pparent contraction of the spatial scales in the direction of motion of the source FitzGerald-Lorentz contraction) proportional to the factor

$$1 - \frac{v_e^2}{2c^2} \approx \sqrt{1 - \frac{v_e^2}{c^2}}$$

hich exactly compensates for the expected second-order effect.

In fact MM result strongly suggests the existence of an absolute medium beause the FitzGerald-Lorentz contraction is a mandatory effect in a field theory based n the nonlinear generalizations of the D'Alembert wave equation (see the comments a preceding section). Thus the soliton paradigm provides a most natural explanaion of the nil effect of MM experiment. If the "material bodies" (e.g., the arms of he interferometer) are *bound states* of solitons, then they are contracted in the diection of motion because the solitons themselves and the inter-soliton distances are. he latter are defined mainly by the electromagnetic forces whose length-scales also uffer contraction. Hence the distance that must be traveled by the light through the *uiescent* metacontinuum between two soliton formations (the emitter and receptor), indeed shorter in the direction of motion in comparison with the path traveled in ransverse direction.

Note that the *metacontinuum* itself is not contracted and that is why the peed of light has a constant value in each coordinate system connected with the ropagating objects—*solitons*.

The real proof of the presence of a *metacontinuum* would be an experiment or the first-order effect. Such an experiment can only be achieved if *two different* ources of light are employed with sufficiently synchronized frequencies. There are vailable in the market lasers with the required stabilization (e.g., the *HeNe* Model 00 of Coherent Components Group) but it goes beyond the frame of the present work to present the details on hardware. As the sought effect is of order of 10^{-4} then if the two sources are synchronized up to 10^{-6}, the accuracy would be of order of 1%. In this sense we will call such sources "identical." Here we give in Fig. 2 the principle scheme of a possible first-order experiment.

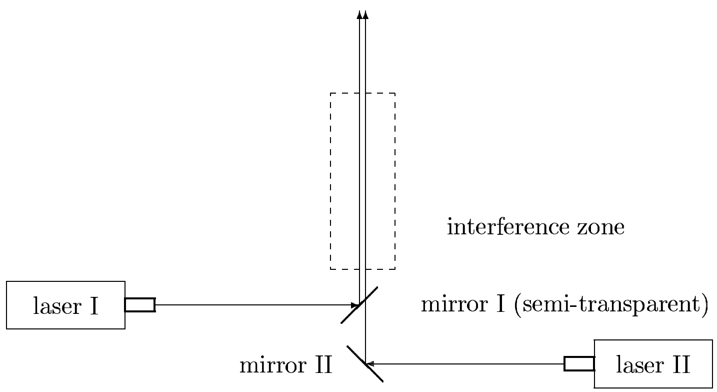

Fig. 2. Principal scheme of the interferometry experiment.

We deliberately exclude from consideration any kind of non-optical experiment and leave beyond our scope the optical experiments in dense matter (water filled columns, etc.). Consider two "identical" sources of monochromatic light which move together in the same direction with the same velocity. The first of them emits a plane wave propagating in the direction of motion of and the second one—in the opposite direction. By means of a mirror and a semi-transparent mirror the two plane waves are made co–linear. The beam of the second laser is reflected by the mirror II changing its direction on 90° and made to pass through a semi-transparent mirror whose reflecting surface serve to change the direction of the beam of first laser on 90°. Beyond the semi-transparent mirror the two beams are parallel and can produce an easily discernible interference stripped pattern. A snapshot of the region of interference would reveal strips of different intensity gradually transforming into each other and the modulation frequency can easily be measured.

It is interesting to note that Jaseda *et al.* [34] already used two lasers in interferometry experiment in order to quantitatively verify the FG–L contraction, but in their experiments the lasers beams were parallel (aiming to verify quantitatively the contraction) while in the proposed here experiment they are anti–parallel since now it is not the contraction that needs verification, but the very existence to the first-order of Doppler effect.

For the sake of self-containedness of the paper we outline here the derivation of Doppler effect (see also [35]). The harmonic waves propagating in presumably

uiescent medium are given by the following formula

$$F_{\pm}(x,t) \equiv e^{i(k_r x \mp \omega_r t)}\ , \quad k_{\pm} = \frac{\omega_r}{c}\ , \quad \lambda_{\pm} = \frac{c}{\omega_r}\ , \tag{5.9}$$

here $\omega_{\pm}$ are the frequencies. The upper sign in the notations refers to the wave ropagating in positive direction, while the lower sign—to the wave propagating in egative direction. These waves have to satisfy the boundary condition on the moving oundaries (the sources)

$$F_{\pm}(\pm v_e t, t) \equiv e^{i\omega_0 t}\ , \tag{5.10}$$

here v_e is the velocity of the moving frame relatively to the metacontinuum. Re-pectively, if the sources were at rest, then they would have produced waves with ave number $k_0 = \omega_0/c$ and wave length $\lambda_0 = k_0^{-1}$. The boundary condition (5.10) ields the following relation for the parameters of the propagating wave:

$$\omega_{\pm} = \omega_0 \left(1 \mp \frac{v_e}{c}\right)^{-1} \quad k_{\pm} = \frac{\omega_0}{c}\left(1 \mp \frac{v_e}{c}\right)^{-1} \quad \lambda_{\pm} = \lambda_0 \left(1 \mp \frac{v_e}{c}\right)\ . \tag{5.11}$$

The role of the mirrors is to change the direction of propagation of each ave without destroying its plane nature. After the reflection, the two waves are ropagating as planar waves in the positive direction of z-axis (vertical in Fig. 2): $_{\pm}(z,t) \equiv e^{i(k_{\pm} z - \omega_{\pm} t)}$. Then in the interference region one has a wave which is the uperposition of two of them for a given moment of time (say, $t = 0$) so that

$$\Re\left[F_+(z,t) + F_-(z,t)\right] = 2\cos\left(\frac{k_+ + k_-}{2} z\right)\cos\left(\frac{k_+ - k_-}{2} z\right)\ , \tag{5.12}$$

hich is a modulated wave with a wave number of the carrier $\frac{1}{2}(k_+ + k_-) = k_0 + O(d^2)$ nd with a wave number for the modulation $\frac{1}{2}(k_+ - k_-) = dk_0 + O(d^3)$. Respectively he expressions for wave lengths valid to the second order, are λ_0 and $\lambda_m = \lambda_0 d^{-1}$. or red-light lasers the length of the wave is $\lambda_0 \approx 6.3 \cdot 10^{-5}$ cm and then for the length f modulation wave one has $\lambda_m = 0.63$ cm and the strips produced must be easily etectable on an optical table of standard dimensions.

Alternative way around is to look for the fringes formed on the semi-transparent nirror. The length scale of a fringe would be around 6.3 cm for the red light and the aser beams should be expanded so as to cover a region of 40-50 cm when they reach he semi-transparent mirror.

6. Quasi, Pseudo, Meta: Concluding Remarks

From the point of view of soliton paradigm, the notion of discrete versus continuous (wave–particle dualism) is revisited. As a featuring example a very thin layer of Hookean elastic medium of very large dilational modulus is considered: a special kind of N-dimensional shell called *gossamer*. It is shown that the linearized equations for the laminar displacements have as a corollary the Maxwell equations for appropriately defined quantities called electric and magnetic fields. A higher-order dispersive and nonlinear Boussinesq equation is derived for the flexural deformations of *gossamer* ("master" equation of the wave mechanics). Its linear part is the linearized Schrödinger equation. Due to the conservative properties of the "master" equation, the flexural solitary waves (*flexons*) appear to be solitons. We call "Soliton Paradigm" the conceptual framework in which the solitons are identified as the particles, i.e. there is no dichotomy between particles and waves. A *particle* (or *corpuscles*) is a notion to signify our perception of the geometry and dynamics of a localized wave ("lump" of deformations, stress, energy, etc.) of *metacontinuum*. The Hamiltonian properties of wave-mechanics equation (the *metadynamics*) define the Hamiltonian (Newtonian) dynamics of the phase objects (*particles-solitons*). Thus the de-Broglie wave-particle duality is a matter of observation and perception: we are faced with a unique object—a localized wave, which is perceived either as a corpuscular object (in fact its center of wave-mass) or as a wave. Particles-Solitons are not moving *through* the metacontinuum, rather they are *phase patterns* propagating *over* it without disturbing its contiguity. Hence no aether-drift effect will be associated with the motion of a particle-soliton. A possible experimental set-up is proposed for verification of this concept.

Solutions of the "master" equation (called *flexons*) are obtained numerically by means of Method of Variational Imbedding. Localized torsional structures of integer-valued topological charge are discussed qualitatively and identified as *charges*. It is shown that the membrane tension in the *gossamer* creates attractive force (*gravitation*) between the large ensembles of localized flexural deflections-particles which is proportional to r^{1-N}.

The concept of unification based on *metacontinuum* and *soliton paradigm* gives:

- Maxwell Equations for the stress interactions in the middle surface of the *gossamer*;
- Dispersive equation (Schrödinger's Schrödinger equation) for the wave function of the transverse flexural deformations alongside the fourth spatial dimension;
- Gravitation as membrane (*anti*) tension in the middle surface;
- Charges as vortex-like solutions.

and can be summarized as follows

Metacontinuum
$\Downarrow$
Particles (Localized Waves — Solitons)
and *Fields* (Derivations of Stress- and Strain-fields of *metacontinuum*)
$\Downarrow$
Hamiltonian-Lagrangian Formalism
$\Downarrow$
Newtonian mechanics of "point particles" (centers of *solitons*)
$\Downarrow$
Mechanical continua (approximation for ensembles of particles)

The unification here is not only for the forces of interaction (the different elds), but it also fuses the Wave-particle duality into Particle-Wave *unity* subordiating the concept of a *particle* (*corpuscle*) to the one of localized phase patterns ngendered by the balance between the membrane anti-tension and the nonlinearity nd dispersion of the "master" equation of wave mechanics.

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