Math 270–005: Calculus I
Prof. Arturo Magidin
Homework 12
SOLUTIONS

§4.9

23. We have
$$\int (3x^5 - 5x^9)\,dx = \frac{1}{2}x^6 - \frac{1}{2}x^{10} + C.$$
We can verify by differentiation:
$$\left(\frac{1}{2}x^6 - \frac{1}{2}x^{10} + C\right)' = \frac{6}{2}x^5 - \frac{10}{2}x^9 = 3x^5 - 5x^9.$$

31. We do the product and then the integral:
$$\begin{aligned}\int (3x+1)(4-x)\,dx = \int (12x + 4 - 3x^2 - x)\,dx &= \int (-3x^2 + 11x + 4)\,dx\\ &= -x^3 - \frac{11}{2}x^2 + 4x + C.\end{aligned}$$
Indeed, we have:
$$\left(-x^3 - \frac{11}{2}x^2 + 4x + C\right)' = -3x^2 - \frac{22}{2}x + 4 = -3x^2 - 11x + 4.$$

39. Here we have:
$$\int (\csc^2\theta + 2\theta^2 - 3\theta)\,d\theta = -\cot\theta + \frac{2}{3}\theta^3 - \frac{3}{2}\theta^2 + C.$$
Verifying:
$$\frac{d}{d\theta}\left(-\cot\theta + \frac{2}{3}\theta^3 - \frac{3}{2}\theta^2 + C\right) = -(\csc^2\theta) + \frac{6}{3}\theta^2 - \frac{6}{2}\theta + C = \csc^2\theta + 2\theta^2 - 3\theta.$$

51. Here we have:
$$\int \frac{1}{2y}\,dy = \frac{1}{2}\int \frac{1}{y}\,dy = \frac{1}{2}\ln|y| + C.$$
Indeed,
$$\frac{d}{dy}\left(\frac{1}{2}\ln|y| + C\right) = \frac{1}{2}\left(\frac{1}{y}\right) = \frac{1}{2y}.$$

69. The general antiderivative of $f(x) = x^5 - 2x^2 + 1$ is
$$\int (x^5 - 2x^2 + 1)\,dx = \frac{1}{6}x^6 - \frac{2}{3}x^3 + x + C.$$
If we want $F(0) = 1$, then we want $C = 1$. So the desired antiderivative is
$$F(x) + \frac{1}{6}x^6 - \frac{2}{3}x^3 + x + 1.$$

77. If $f'(x) = 2x - 3$ and $f(0) = 4$, then

$$f(x) = \int (2x-3)\,dx = x^2 - 3x + C.$$

Since $f(0) = 4$, then $C = 4$, so $f(x) = x^2 - 3x + 4$.

§5.4

25. Since x^3 is an odd function, we have that $\int_{-1}^{1} x^3\,dx = 0$. So the average value of $f(x)$ is

$$\overline{f} = \frac{1}{1-(-1)} \int_{-1}^{1} x^3\,dx = \frac{1}{2}(0) = 0.$$

Alternatively, if you do not realize we are dealing with an odd function, we have:

$$\overline{f} = \frac{1}{1-(-1)} \int_{-1}^{1} x^3\,dx = \frac{1}{2}\left(\frac{1}{4}x^4 \Big|_{-1}^{1}\right) = \frac{1}{2}\left(\frac{1}{4} - \frac{1}{4}\right) = 0.$$

Here is a picture:

27. The average value of $f(x) = \frac{1}{x^2+1}$ on $[-1, 1]$ is:

$$\overline{f} = \frac{1}{1-(-1)} \int_{-1}^{1} \frac{1}{x^2+1}\,dx = \frac{1}{2}\left(\arctan(x)\Big|_{-1}^{1}\right) = \frac{1}{2}(\arctan(1) - \arctan(-1)) = \frac{1}{2}\left(\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\right) = \frac{\pi}{4}.$$

Here is a picture:

§5.5

17. Letting $u = x^2 - 1$ so that $du = 2x\,dx$, we have:

$$\int 2x(x^2-1)^{99}\,dx = \int u^{99}\,du = \frac{1}{100}u^{100} + C = \frac{1}{100}(x^2-1)^{100} + C.$$

19. Let $u = 1 - 4x^3$. Then $du = -12x^2\,dx$, so $2x^2 = -\frac{1}{6}\,du$. Therefore,

$$\int \frac{2x^2}{\sqrt{1-4x^3}}\,dx = \int \frac{-\frac{1}{6}\,du}{\sqrt{u}} = -\frac{1}{6}\int u^{-1/2}\,du = -\frac{1}{6}\left(2u^{1/2}\right) = -\frac{1}{3}\sqrt{1-4x^3} + C.$$

27. Let $u = x^3$. Then $du = 3x^2\,dx$, so $6x^2\,dx = 2\,du$. Therefore

$$\int 6x^2 4^{x^3}\,dx = \int 4^u 2\,du = 2\int 4^u\,du = 2\left(\frac{1}{\ln(4)}4^u\right) + C = \frac{2}{\ln(4)}4^{x^3} + C = \frac{4^{x^3}}{\ln(2)} + C.$$

The last part because $\ln(4) = \ln(2^2) = 2\ln(2)$.

37. Let $u = 10x + 7$. Then $du = 10\,dx$, so $dx = \frac{1}{10}\,du$. We get:

$$\int \sec^2(10x+7)\,dx = \int \frac{1}{10}\sec^2(u)\,du = \frac{1}{10}\tan(u) + C = \frac{1}{10}\tan(10x+7) + C.$$

47. If we let $u = 4 - x^2$, then $du = -2x\,dx$, so $2x\,dx = (-1)\,du$. When $x = 0$, we have $u = 4$; when $x = 1$, we have $u = 3$. So

$$\begin{aligned}\int_0^1 2x(4-x^2)\,dx &= \int_4^3 u(-1)\,du = -\int_4^3 u\,du \\ &= \int_3^4 u\,du = \frac{1}{2}u^2\,\Big|_3^4 = 8 - \frac{9}{2} = \frac{7}{2}.\end{aligned}$$

57. We let $u = \sin x$; then $du = \cos x\,dx$. When $x = \pi/4$, we have $u = \sqrt{2}/2$. When $x = \pi/2$, we have $u = 1$. So:

$$\begin{aligned}\int_{\pi/4}^{\pi/2} \frac{\cos x}{\sin^2 x}\,dx &= \int_{\sqrt{2}/2}^{1} \frac{du}{u^2} = \int_{\sqrt{2}/2}^{1} u^{-2}\,du \\ &= -u^{-1}\,\Big|_{\sqrt{2}/2}^{1} = -1 - \left(-\frac{1}{\sqrt{2}/2}\right) \\ &= -1 + \frac{2}{\sqrt{2}} = \sqrt{2} - 1.\end{aligned}$$