MATH 270 – Section 005: Calculus I
Spring 2024
DERIVATION FORMULAS AND RULES
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All derivatives are with respect to x.

Basic formulas

- Derivative of a Constant: For any constant c, $(c)' = 0$.
- Power Rule: For any real number r, $(x^r)' = rx^{r-1}$.
- Exponential functions: For any $b > 0$, $b \neq 1$,

$$(e^x)' = e^x$$
$$(b^x)' = \ln(b)b^x.$$

- Logarithms: For any $b > 0$, $b \neq 1$,

$$(\ln|x|)' = \frac{1}{x}$$
$$(\log|x|)' = \frac{1}{\ln(10)x}$$
$$(\log_b|x|)' = \frac{1}{\ln(b)x}$$

- Trigonometric Functions: With angles measured in **radians**:

$$(\sin x)' = \cos(x) \qquad (\cos x)' = -\sin(x)$$
$$(\tan x)' = \sec^2(x) \qquad (\cot x)' = -\csc^2(x)$$
$$(\sec x)' = \sec(x)\tan(x) \qquad (\csc x)' = -\csc(x)\cot(x)$$

- Inverse Trigonometric Functions:

$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \qquad (\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$$
$$(\arctan x)' = \frac{1}{1+x^2} \qquad (\operatorname{arccot} x)' = -\frac{1}{1+x^2}$$
$$(\operatorname{arcsec} x)' = \frac{1}{|x|\sqrt{x^2-1}} \qquad (\operatorname{arccsc} x)' = -\frac{1}{|x|\sqrt{x^2-1}}$$

RULES

- SUM AND DIFFERENCE RULE:

$$(f+g)' = f' + g'$$
$$(f-g)' = f' - g'$$

- CONSTANT MULTIPLE RULE: If c is a constant, $(cf)' = cf'$.
- PRODUCT RULE: $(fg)' = f'g + fg'$.

  That means that:

  - $(fgh)' = f'gh + fg'h + fgh'$
  - $(f_1 f_2 f_3 \cdots f_k)' = f_1' f_2 f_3 \cdots f_k + f_1 f_2' f_3 \cdots f_k + f_1 f_2 f_3' \cdots f_k + \cdots + f_1 f_2 f_3 \cdots f_k'$.

- QUOTIENT RULE: $\left(\dfrac{f}{g}\right)' = \dfrac{gf' - fg'}{g^2}$.

  - In particular: $\left(\dfrac{1}{g}\right)' = -\dfrac{g'}{g^2}$.

- CHAIN RULE: $\Big(f\big(g(x)\big)\Big)' = f'\big(g(x)\big)g'(x)$.

  In particular:

  - $\frac{d}{dx} e^u = e^u u'$
  - $\frac{d}{dx} \sec(u) = \sec(u)\tan(u)u'$
  - $\frac{d}{dx} \ln|u| = \left(\frac{1}{u}\right) u' = \frac{u'}{u}$.
  - $\frac{d}{dx} \arctan(u) = \left(\frac{1}{1+u^2}\right) u' = \frac{u'}{1+u^2}$
  - etc.