Math 270–005: Calculus I
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Homework 4
SOLUTIONS

§3.3

19. If $y = x^5$, then $y' = 5x^4$.

25. If $h(t) = \frac{t^2}{2} + 1$, then $h'(t) = \frac{1}{2}(2t) = t$.

29. If $g(t) = 100t^2$ then $g'(t) = 200t$.

37. If $f(x) = 3e^x + 5x + 5$, then $f'(x) = 3e^x + 5$.

47. If $f(x) = (2x+1)(3x^2+2)$, then to find the derivative (without the product rule), we first expand:

$$\begin{aligned} f(x) &= (2x+1)(3x^2+2)\\ &= 6x^3 + 4x + 3x^2 + 2\\ &= 6x^3 + 3x^2 + 4x + 2.\\ f'(x) &= 18x^2 + 6x + 4. \end{aligned}$$

51. Here we have:

$$\begin{aligned} h(x) &= (x^2+1)^2\\ &= x^4 + 2x^2 + 1\\ h'(x) &= 4x^3 + 4x. \end{aligned}$$

57. Here,

$$\begin{aligned} g(w) &= \frac{e^{2w} + e^w}{e^w}\\ &= \frac{e^w(e^w+1)}{e^w}\\ &= e^w + 1. \end{aligned}$$

So $g'(w) = e^w$.

70. Here we have $f(x) = 3x^2 + 5e^x$. We want to find $f'(x)$, $f''(x)$, and $f'''(x)$. We hve:

$$\begin{aligned} f(x) &= 3x^2 + 5e^x\\ f'(x) &= 6x + 5e^x\\ f''(x) &= 6 + 5e^x\\ f'''(x) &= 5e^x. \end{aligned}$$

§3.4

19. First we use the Product Rule, then we simplify:

$$\begin{aligned} f(x) &= 3x^4(2x^2-1)\\ f'(x) &= (3x^4)'(2x^2-1) + 3x^4(2x^2-1)'\\ &= 12x^3(2x^2-1) + 3x^4(4x)\\ &= 24x^5 - 12x^3 + 12x^5 = 36x^5 - 12x^3. \end{aligned}$$

23. Here we have:

$$\begin{aligned}
f(t) &= t^{5/3}e^t \\
f'(t) &= (t^{5/3})'e^t + t^{5/3}(e^t)' \\
&= \frac{5}{3}t^{\frac{5}{3}-1}e^t + t^{5/3}e^t \\
&= \frac{5}{3}t^{2/3}e^t + t^{5/3}e^t \\
&= t^{2/3}e^t\left(\frac{5}{3} + t\right).
\end{aligned}$$

25. Using the quotient rule, we have:

$$\begin{aligned}
f(x) &= \frac{e^x}{e^x+1} \\
f'(x) &= \frac{(e^x+1)(e^x)' - e^x(e^x+1)'}{(e^x+1)^2} \\
&= \frac{(e^x+1)(e^x) - e^x(e^x)}{(e^x+1)^2} \\
&= \frac{e^{2x} + e^x - e^{2x}}{(e^x+1)^2} \\
&= \frac{e^x}{(e^x+1)^2}.
\end{aligned}$$

33. Product rule:

$$\begin{aligned}
g(w) &= e^w(w^3-1) \\
g'(w) &= (e^w)'(w^3-1) + e^w(w^3-1)' \\
&= e^w(w^3-1) + e^w(3w^2) = e^w(w^3+3w^2-1).
\end{aligned}$$

37. Quotient Rule:

$$\begin{aligned}
g(x) &= \frac{e^x}{x^2-1} \\
g'(x) &= \frac{(x^2-1)(e^x)' - e^x(x^2-1)'}{(x^2-1)^2} \\
&= \frac{(x^2-1)e^x - e^x(2x)}{(x^2-1)^2} \\
&= \frac{e^x(x^2-2x-1)}{(x^2-1)^2}.
\end{aligned}$$

47. A quotient rule that requires a product rule along the way:

$$\begin{aligned} h(x) &= \frac{xe^x}{x+1} \\ h'(x) &= \frac{(x+1)(xe^x)' - (xe^x)(x+1)'}{(x+1)^2} \\ &= \frac{(x+1)\big((x)'e^x + x(e^x)'\big) - xe^x}{(x+1)^2} \\ &= \frac{(x+1)(e^x + xe^x) - xe^x}{(x+1)^2} \\ &= \frac{xe^x + x^2e^x + e^x + xe^x - xe^x}{(x+1)^2} \\ &= \frac{(x^2+x+1)e^x}{(x+1)^2} \end{aligned}$$

§3.5

11. We have

$$\lim_{x\to 0} \frac{\sin(3x)}{x} = \lim_{x\to 0} \frac{3\sin(3x)}{3x} = 3\lim_{x\to 0} \frac{\sin(3x)}{3x} = 3(1) = 3.$$

13. Similarly,

$$\lim_{x\to 0} \frac{\sin 7x}{\sin 3x} = \lim_{x\to 0} \frac{\frac{7\sin 7x}{7x}}{\frac{3\sin 3x}{3x}} = \frac{7}{3}\left(\frac{\lim_{x\to 0} \frac{\sin 7x}{7x}}{\lim_{x\to 0} \frac{\sin 3x}{3x}}\right) = \frac{7}{3}\left(\frac{1}{1}\right) = \frac{7}{3}.$$

23. If $y = \sin x + \cos x$, then $y' = \cos x - \sin x$.

25. If $y = e^{-x}\sin x = \frac{\sin x}{e^x}$, we have (using the quotient rule):

$$\begin{aligned} y' &= \frac{e^x(\sin x)' - (\sin x)(e^x)'}{(e^x)^2} = \frac{e^x\cos x - e^x\sin x}{(e^x)^2} \\ &= \frac{e^x(\cos x - \sin x)}{(e^x)^2} = \frac{\cos x - \sin x}{e^x} = e^{-x}(\cos x - \sin x). \end{aligned}$$

31. If $y = \sin x\cos x$, then

$$y' = (\sin x)'\cos x + \sin x(\cos x)' = \cos x\cos x - \sin x\sin x = \cos^2 x - \sin^2 x.$$

If you want to be fancy, you can also note that $\cos^2 x - \sin^2 x = \cos(2x)$.

33. If $y = \cos^2 x = \cos x\cos x$, then $y' = (\cos x)'\cos x + \cos x(\cos x)' = -2\sin x\cos x$. If you want to be fancy, note that $2\sin x\cos x = \sin(2x)$, so we could rewrite this as $y' = -\sin(2x)$.

43. If $y = \sec x + \csc x$, then $y' = \sec x\tan x - \csc x\cot x$.

45. If $y = e^x\csc x$, then

$$y' = (e^x)'\csc x + e^x(\csc x)' = e^x\csc x - e^x\csc x\cot x = e^x\csc x(1 - \cot x).$$

57. If $y = x \sin x$, then

$$\begin{aligned} y &= x \sin x \\ y' &= (x)' \sin x + x(\sin x)' = \sin x + x \cos x \\ y'' &= (\sin x)' + (x)' \cos x + x(\cos x)' \\ &= \cos x + \cos x - x \sin x \\ &= 2 \cos x - x \sin x. \end{aligned}$$