Math 270–005: Calculus I
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Homework 3
SOLUTIONS

§3.1

13. If $s(t) = -16t^2 + 100t$ is the position at time t, then the instantaneous velocity at time $a = 1$ is:

$$\begin{aligned} s'(1) &= \lim_{t\to 1} \frac{s(t)-s(1)}{t-1} = \lim_{t\to 1}\frac{(-16t^2+100t)-84}{t-1}\\ &= \lim_{t\to 1}\frac{-16t^2+100t-84}{t-1} = \lim_{t\to 1}\frac{(t-1)(-16t+84)}{t-1}\\ &= \lim_{t\to 1}\frac{-16t+84}{1} = -16+84 = 68. \end{aligned}$$

So the velocity at $t = 1$ is 68 feet/sec.

15. The function is $f(x) = x^2 - 5$; the point is $P(3,4)$ (note that, indeed, $f(3) = 4$).

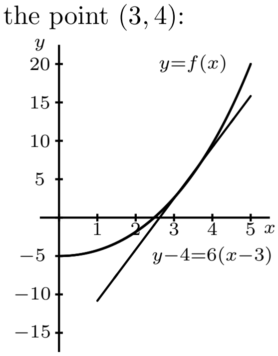
(a) Using Definition 1, we have that the slope is:

$$\begin{aligned} m_{\tan} &= \lim_{x\to 3}\frac{f(x)-f(3)}{x-3} = \lim_{x\to 3}\frac{(x^2-5)-4}{x-3}\\ &= \lim_{x\to 3}\frac{x^2-9}{x-3} = \lim_{x\to 3}\frac{(x-3)(x+3)}{x-3}\\ &= \lim_{x\to 3}\frac{x+3}{1} = 3+3 = 6. \end{aligned}$$

(b) The equation of the tangent is

$$\begin{aligned} y - f(a) &= m_{\tan}(x-a)\\ y - 4 &= 6(x-3). \end{aligned}$$

(c) Here is a plot of the region near the point $(3,4)$:



21. The function is $f(x) = 2x + 1$, the point is $(0,1)$: note that $f(0) = 1$.

(a) Here we have, using Definition (2),

$$\begin{aligned} m_{\tan} &= \lim_{h\to 0}\frac{f(0+h)-f(0)}{h} = \lim_{x\to 0}\frac{f(h)-f(0)}{h}\\ &= \lim_{h\to 0}\frac{(2h+1)-1}{h} = \lim_{h\to 0}\frac{2h}{h}\\ &= \lim_{h\to 0}\frac{2}{1} = 2. \end{aligned}$$

(b) The equation of the tangent is given by

$$\begin{aligned} y - f(a) &= m_{\tan}(x-a) \\ y - 1 &= 2(x-0) \\ y - 1 &= 2x. \end{aligned}$$

35. Here $f(x) = 4x^2 + 2x$, with $a = -2$.

(a) We want to find $f'(a)$. We have:

$$\begin{aligned} f'(-2) &= \lim_{h\to 0} \frac{f(-2+h) - f(-2)}{h} = \lim_{h\to 0} \frac{4(h-2)^2 + 2(h-2) - (16-4)}{h} \\ &= \lim_{h\to 0} \frac{4(h^2 - 4h + 4) + 2h - 4 - 12}{h} = \lim_{h\to 0} \frac{4h^2 - 16h + 16 + 2h - 4 - 12}{h} \\ &= \lim_{h\to 0} \frac{4h^2 - 14h}{h} = \lim_{h\to 0} \frac{h(4h-14)}{h} \\ &= \lim_{h\to 0} \frac{4h-14}{1} = -14. \end{aligned}$$

(b) The tangent goes through $(a, f(a)) = (-2, 12)$, with slope -14 So the equation is:

$$y - 12 = -14(x+2).$$

§3.2

21. (a) The function $f(x) = 5x + 2$ has derivative:

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{(5(x+h)+2) - (5x+2)}{h} = \lim_{h\to 0} \frac{5x + 5h + 2 - 5x - 2}{h} \\ &= \lim_{h\to 0} \frac{5h}{h} = \lim_{h\to 0} \frac{5}{1} = 5. \end{aligned}$$

So $f'(x) = 5$.

(b) That means that $f'(1) = 5$ and $f'(2) = 5$.

23. (a) Here we have $f(x) = 4x^2 + 1$. We have:

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{(4(x+h)^2 + 1) - (4x^2+1)}{h} \\ &= \lim_{h\to 0} \frac{4(x^2 + 2xh + h^2) + 1 - 4x^2 - 1}{h} = \lim_{h\to 0} \frac{4x^2 + 8xh + 4h^2 - 4x^2}{h} \\ &= \lim_{h\to 0} \frac{8xh + 4h^2}{h} = \lim_{h\to 0} \frac{h(8x+4h)}{h} \\ &= \lim_{h\to 0} \frac{8x+4h}{1} = 8x. \end{aligned}$$

So $f'(x) = 8x$.

(b) So we have $f'(2) = 16$ and $f'(4) = 32$.

35. (a) The derivative of $f(x) = 3x^2 + 2x - 10$ is

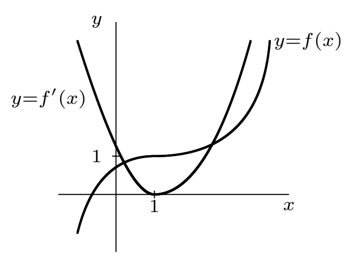
$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{(3(x+h)^2 + 2(x+h) - 10) - (3x^2 + 2x - 10)}{h}$$
$$= \lim_{h\to 0} \frac{3x^2 + 6xh + 3h^2 + 2x + 2h - 10 - 3x^2 - 2x + 10}{h} = \lim_{h\to 0} \frac{6xh + 3h^2 + 2h}{h}$$
$$= \lim_{h\to 0} \frac{h(6x + 3h + 2)}{h} = \lim_{h\to 0} \frac{6x + 3h + 2}{1} = 6x + 2.$$

So $f'(x) = 6x + 2$.

(b) At $a = 1$, we have $f(1) = 3 + 2 - 10 = -5$ and $f'(1) = 6 + 2 = 8$; so the tangent line goes through $(1, -5)$ and has slope 8. Therefore, an equation for the tangent line to the graph at $a = 1$ is

$$y = 8(x - 1) + (-5)$$
$$y = 8(x - 1) - 5$$
$$y = 8x - 13.$$

49. See the book for the graph. A sketch of the function and its derivative is:



To explain: moving left to right, the slope begins large and positive, then comes down towards 0, and is 0 around $x = 1$. Then it starts rising again.

53. See the book for the graph.

(a) The only value of x in $(0, 3)$ where f is not continuous is $x = 1$.

(b) The values of $f(x)$ in $(0, 3)$ where f is differentiable are $x = 1$ (where it is not continuous), and $x = 2$ (where it has a corner).

(c) Here is a sketch of the graph of $f'(x)$: from 0 to 1, it starts near 0 and rises to about 1. From 1 to 2 it seems to drop from about 0 to about -1. And from 2 onward it is constant at about 1: